

Introduction to Medical Image Segmentation

HST 582

Matthew Toews

(slides adapted from William Wells III)

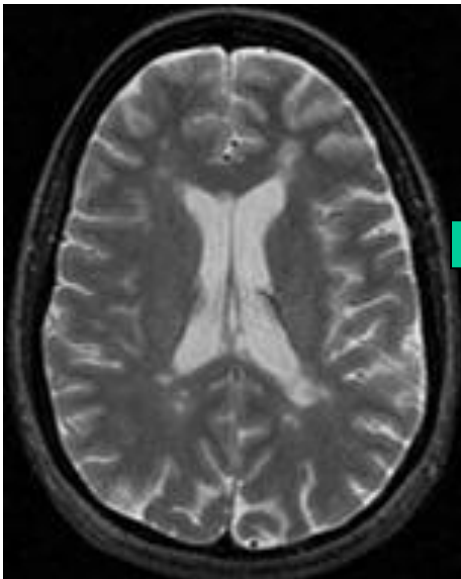
Outline

- Terminology & Applications
- Probability Review
- Intensity-Based Classification
- Prior models
- Morphological Operators

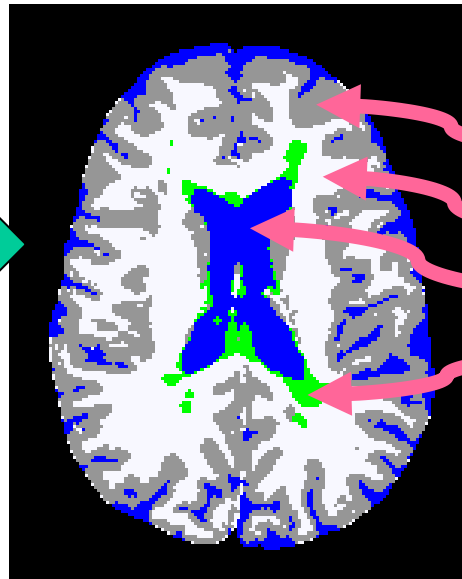
Image Segmentation

Partitioning an image into regions defined by pixel intensity and geometry.

Input: Brain MRI



Output: Pixel Labels



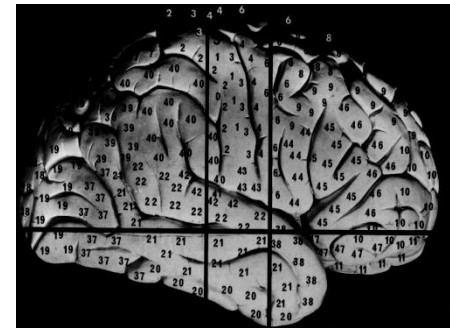
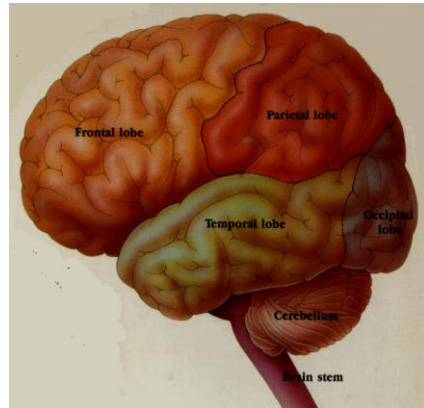
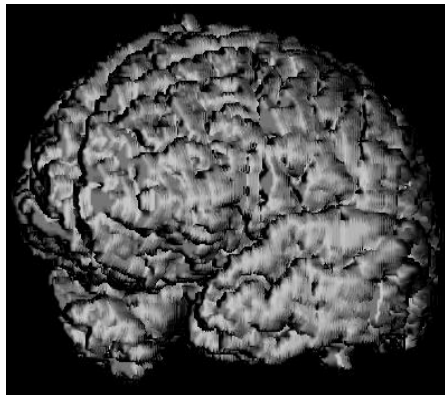
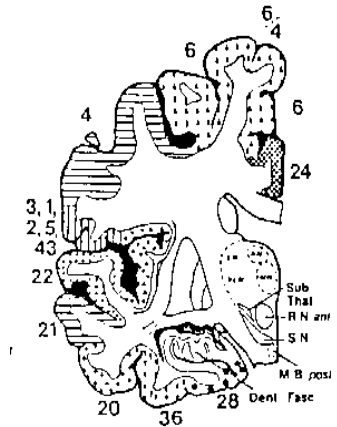
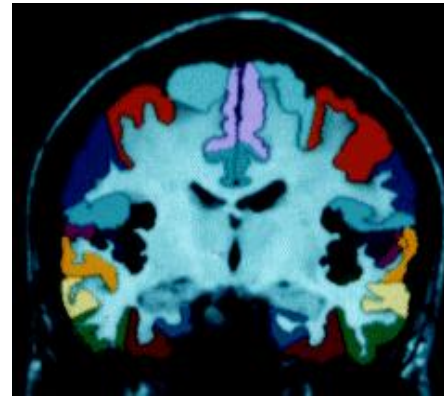
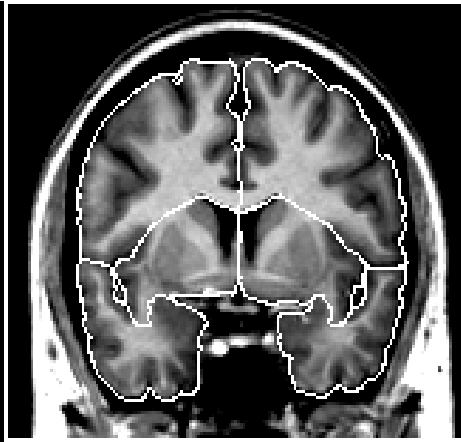
Grey Matter (GM)

White Matter (WM)

Cerebrospinal Fluid (CSF)

Multiple Sclerosis Lesions

Anatomical Description Hierarchy



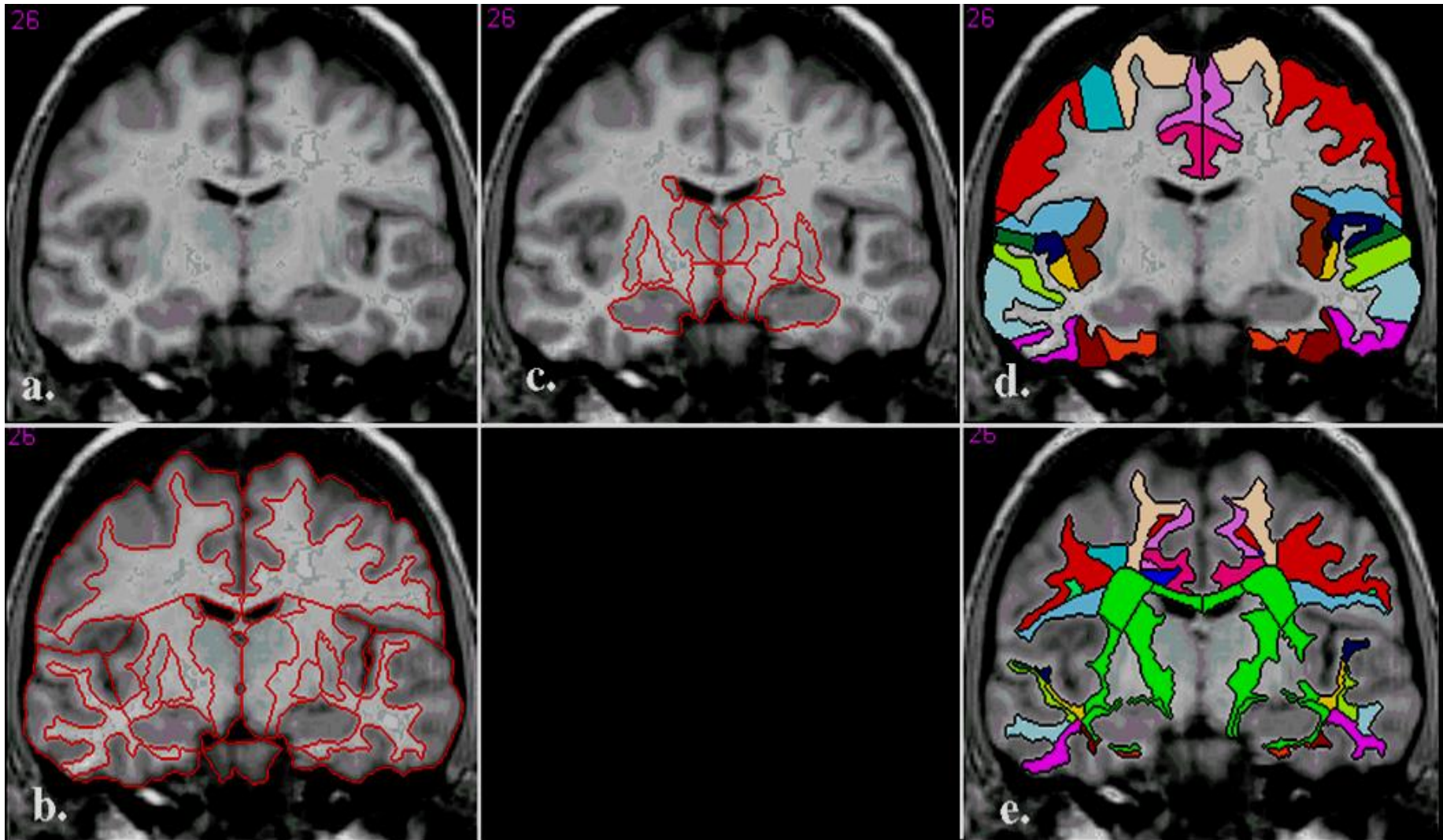
**WHOLE
BRAIN/STRUCTURE**

LOBES

SYSTEMS

**CYTO- & MYELO-
ARCHITECTURE**

Stages of Anatomic Analysis

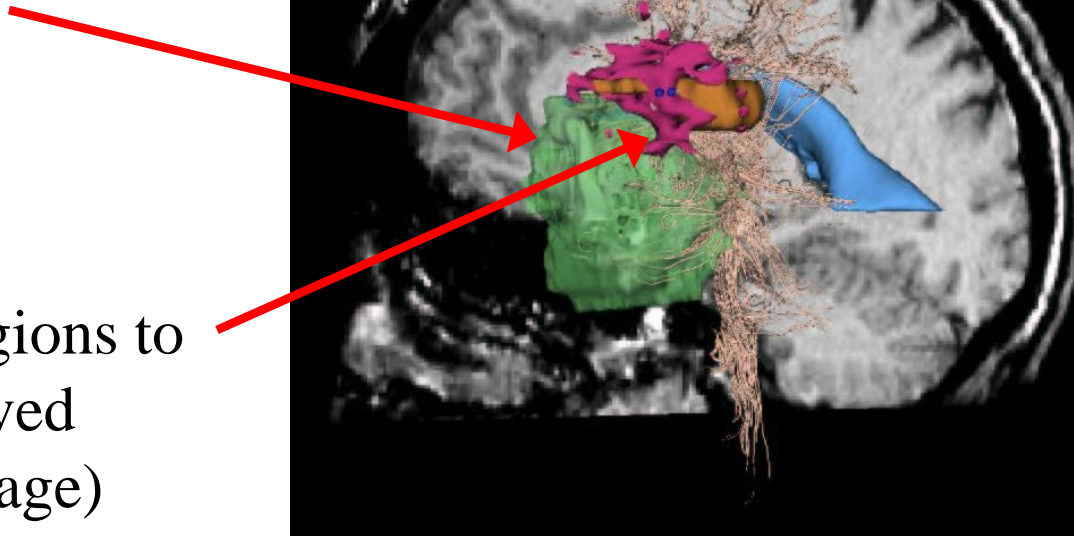


Medical Applications of Segmentation

- Image Guided Surgery
- Surgical Simulation
- Neuroscience Studies
- Therapy Evaluation

fMRI and Electro Corticography

Tumor to be resected



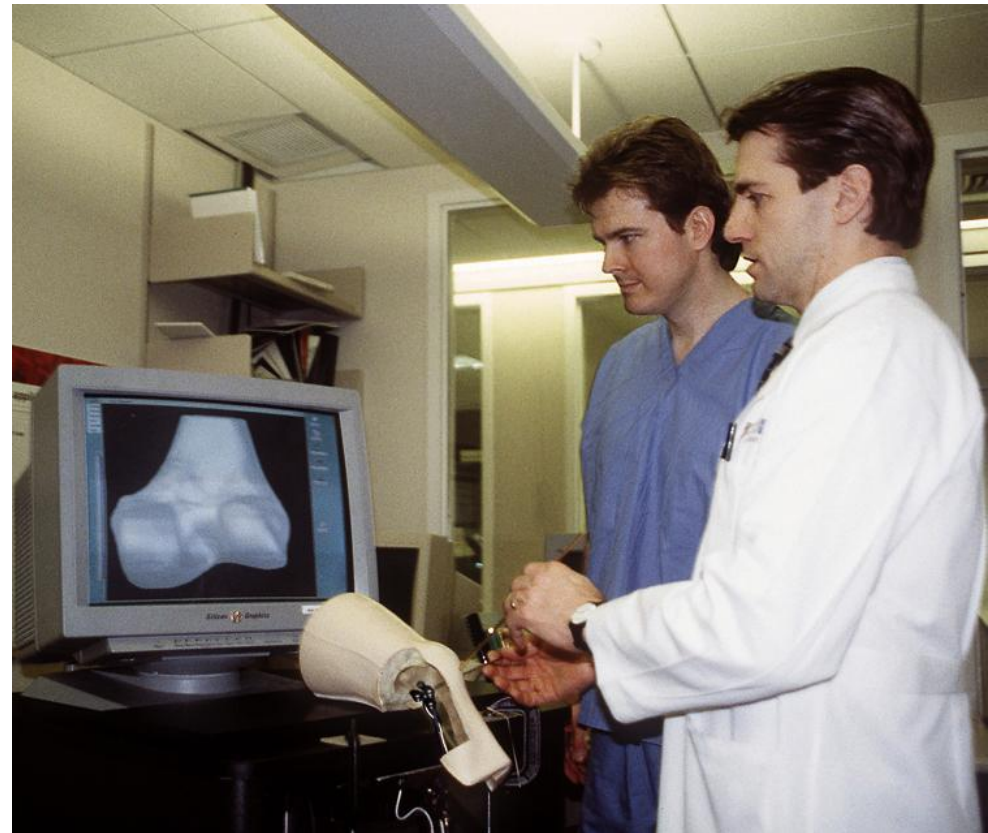
Functional regions to be preserved (e.g. language)

- **Quantitative comparison of functional MRI and direct electrocortical stimulation for functional mapping.** Larsen S, Kikinis R, Talos IF, Weinstein D, Wells W, Golby A. Int J Med Robot. 2007 Sep;3(3):262-70.

Applications of Segmentation

- Image Guided Surgery
- Surgical Simulation

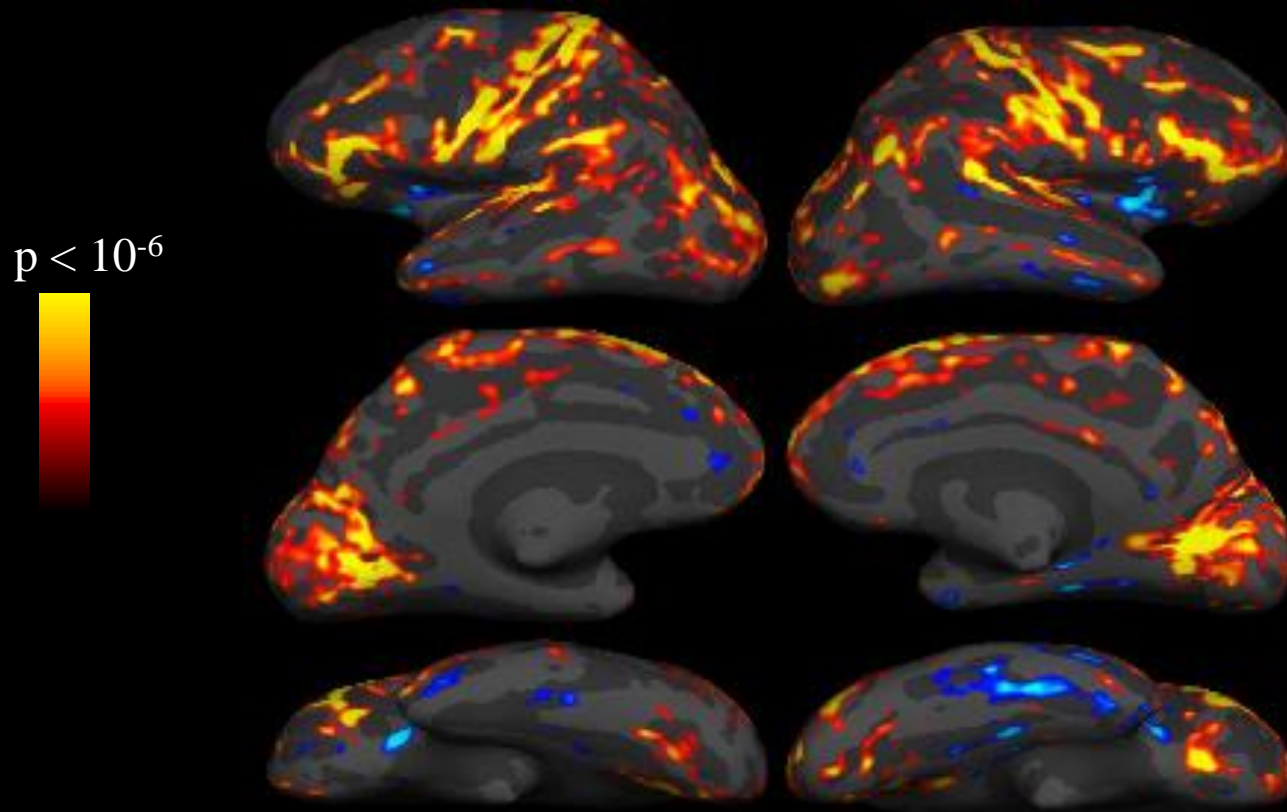
Personalized orthopedic surgery



Applications of Segmentation

- Neuroscience Studies

Statistical Map of Cortical Thinning: Aging

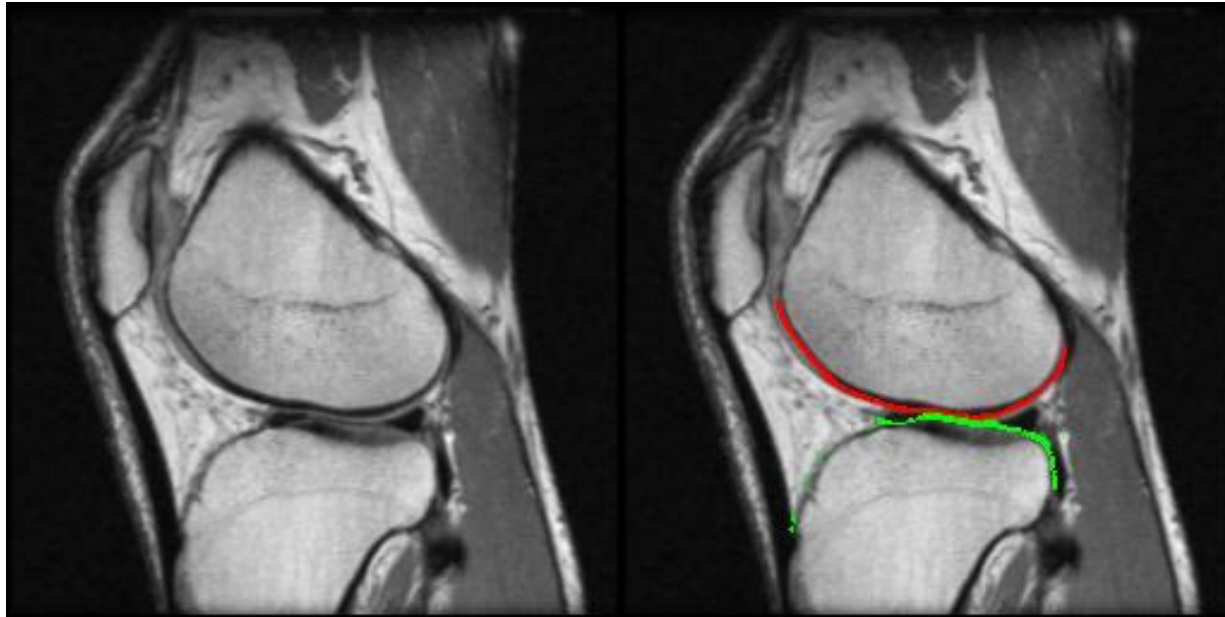


Thanks to Drs. Randy Buckner and David Salat for supplying this slide.
Provided by Bruce Fischl

Applications of Segmentation

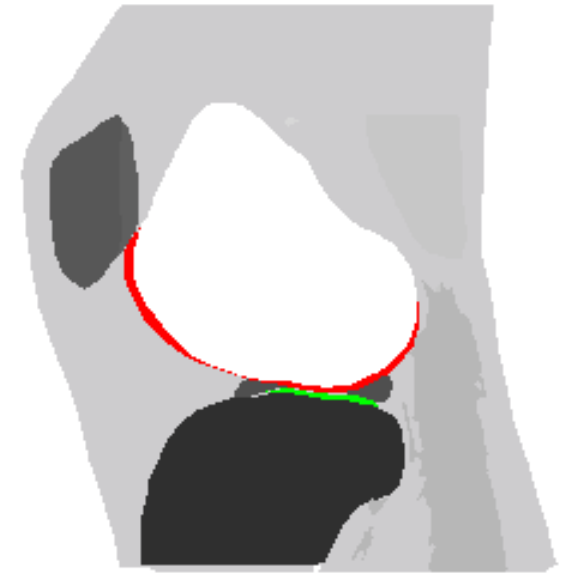
- Therapy Evaluation
 - Multiple Sclerosis
 - Knee Cartilage Repair

Results: Segmentation of Femoral & Tibial Cartilage



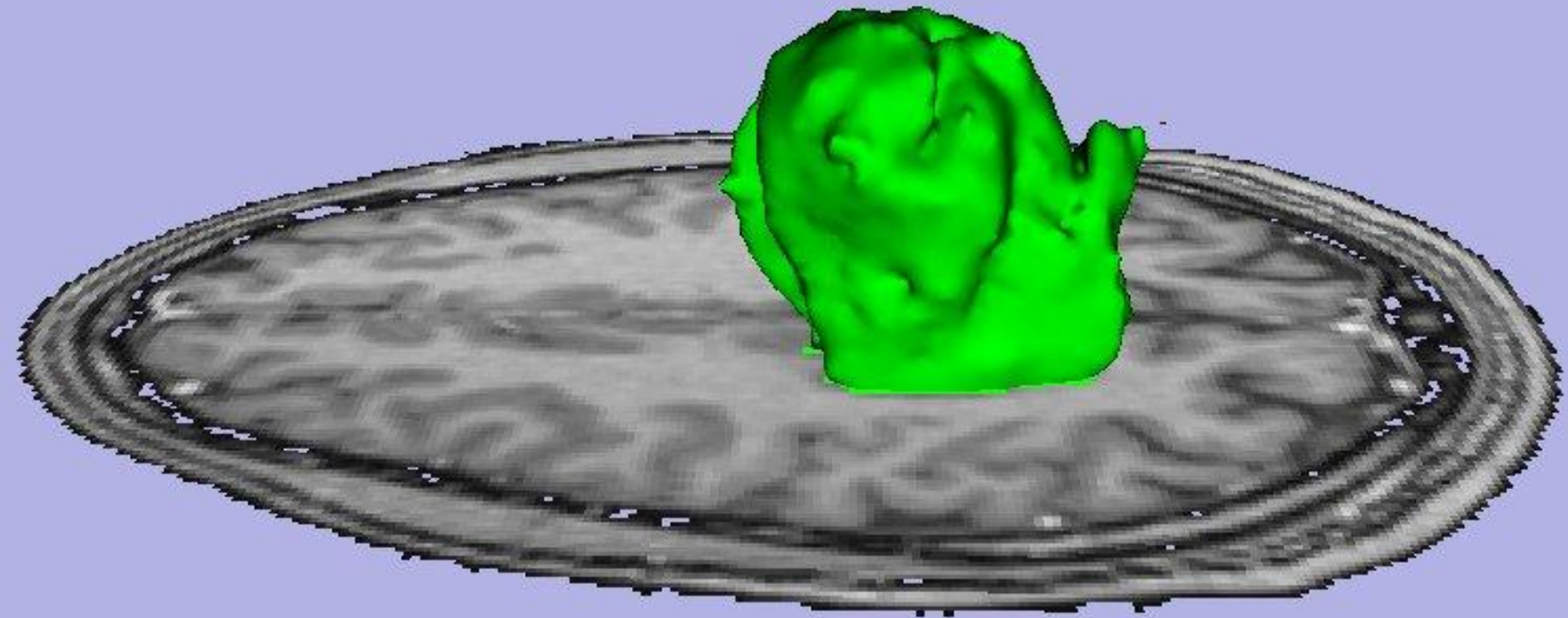
MRI Image

Model-Based
Segmentation



Manual Segmentation

Manual Segmentation



Limitations of Manual Segmentation

- slow (up to 60 hours per scan)
- variable (up to 15% between experts)

[Warfield + 2000]

Automatic Segmentation

An automated segmentation method needs to reconcile

- Gray-level appearance of tissue
 - Characteristics of imaging modality
- Geometry of anatomy

Terminology: *Segmentation*

- HST 582:
 - Labeling images according to tissue type (e.g. White / Gray Matter)
 - Can include ‘object detection’, e.g. a brain region in an image
- Graphics Community:
 - Any process that turns images into models

Probability Review

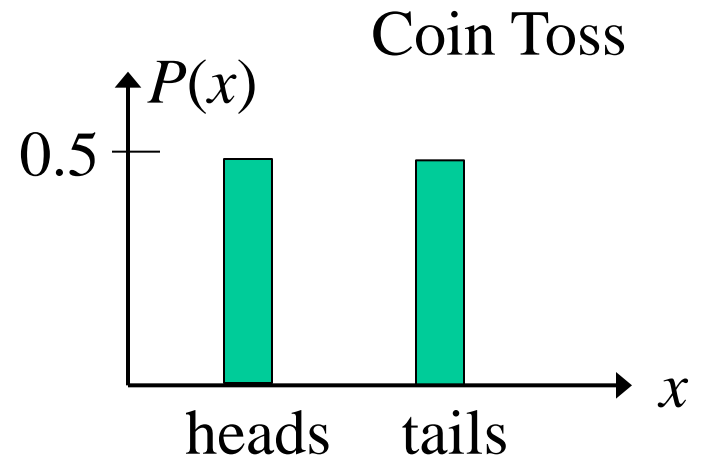
- Discrete Random Variables (RV)
 - Probability Mass Functions (PMF)
- Continuous Random Variables
 - Cumulative Distribution Functions (CDF)
 - Probability Density Functions (PDF)
- Conditional Probability
- Bayes' Rule

Discrete Random Variable

- Characterized by *Probability Mass Function* (PMF)
 - (sometimes called Distribution)
 - Maps values x to their Probabilities $P(x)$

$$0 \leq P(x) \leq 1$$

$$\sum_x P(x) = 1$$



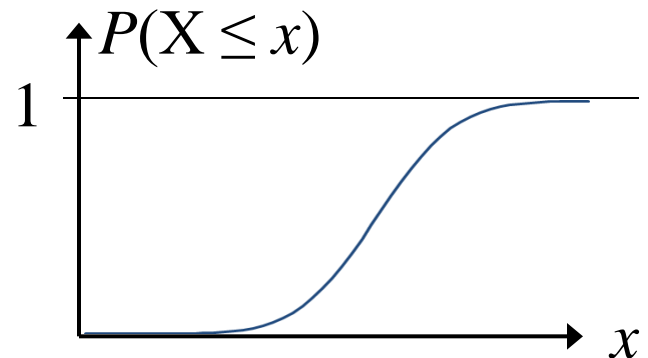
Continuous Random Variables

- Define Cumulative Distribution Function (CDF) on RV \mathbf{x}

$$F_X(x) = P(X \leq x)$$

$$0 \leq F_X(x) \leq 1$$

- Non-Decreasing
- Sometimes called *Distribution Function*



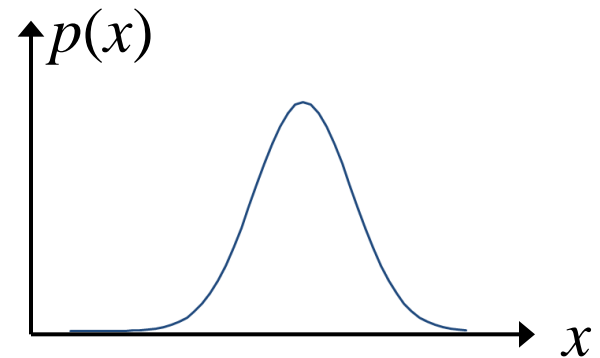
Continuous Random Variables...

- Define *Probability Density Function* (PDF)

$$p(x) = \frac{d}{dx}F_X(x)$$

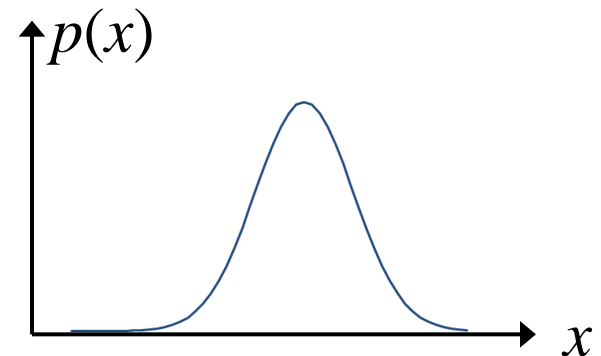
- Easy to show, using Fundamental Theorem of Calculus:

$$P(a \leq x \leq b) = \int_a^b p(x)dx$$



More on PDFs : $p(x)$

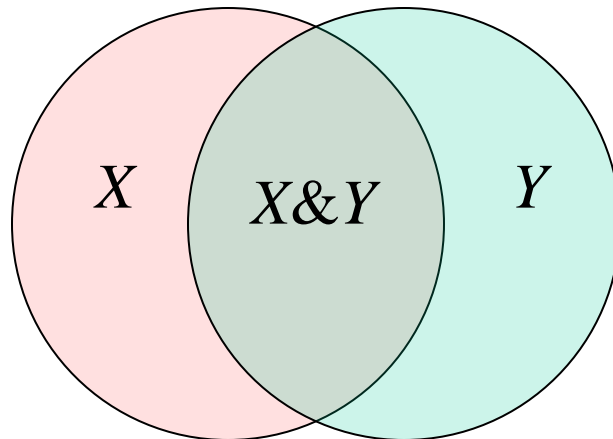
- Non Negative
- Integrates to One
- (Value can be Greater than One)



Conditional Probability

- Define Conditional Probability:

$$P(X|Y) = \frac{P(X \& Y)}{P(Y)}$$



Bayes' Rule (easy to show)

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- Frequent Situation:
 - A : State of the World
 - B : Measurement, Observation
 - $P(B/A)$: Measurement Model
 - $P(A)$: A-Priori Model

Intensity-Based Segmentation

- Statistical Classification
 - ML
 - MAP, a-priori models

Segmentation

- Easy Segmentation
 - Tissue/Air (except bone in MR)
 - Bone in CT
- Feasible Segmentation
 - White Matter/Gray Matter
 - M.S. Lesions

Statistical Classification

- Probabilistic model of intensity as a function of (tissue) class
- Intensity data
- Prior model



Classification of voxels

[Duda, Hart 78]

Measurement Model

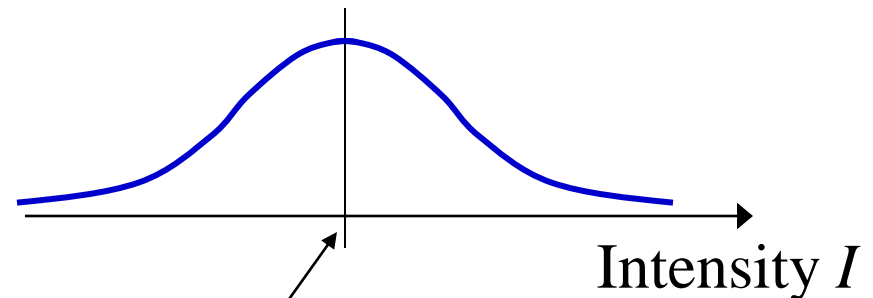
- Characterize sensor

$$p(I|\text{Tissue class } J)$$

probability density

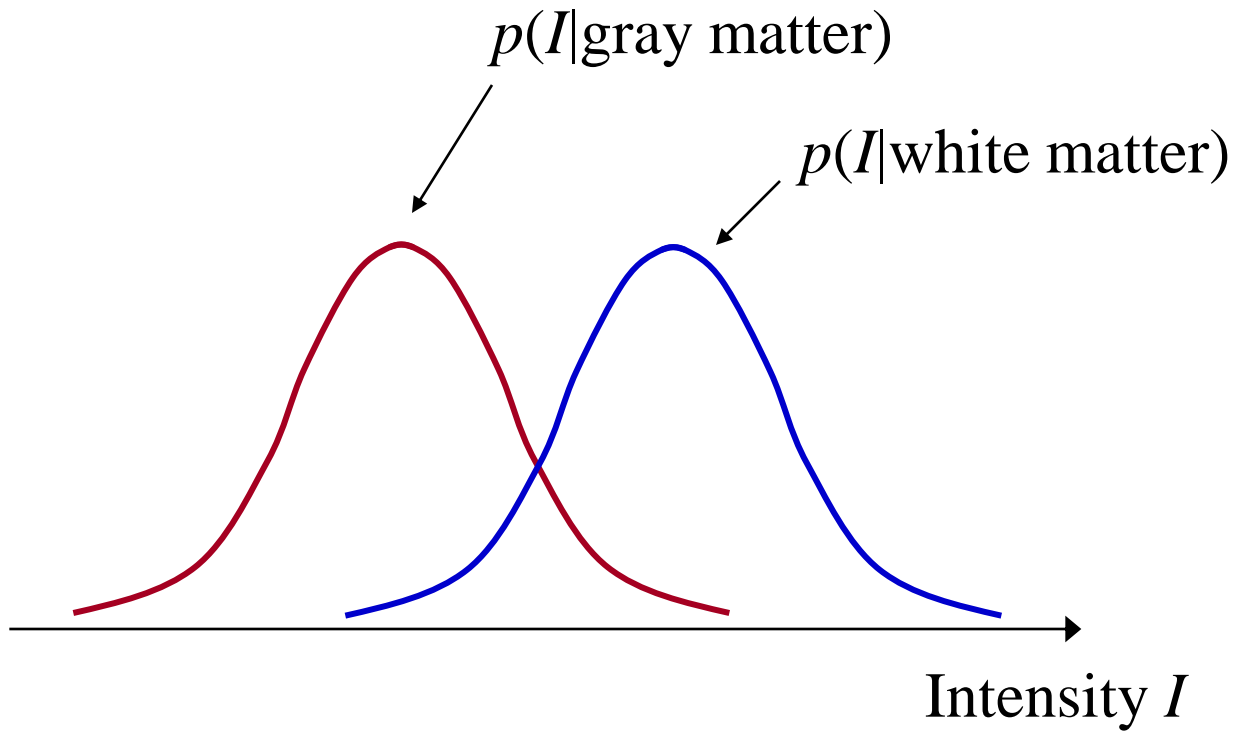
intensity

Tissue class conditional model
of signal intensity



Mean intensity of tissue J

Example



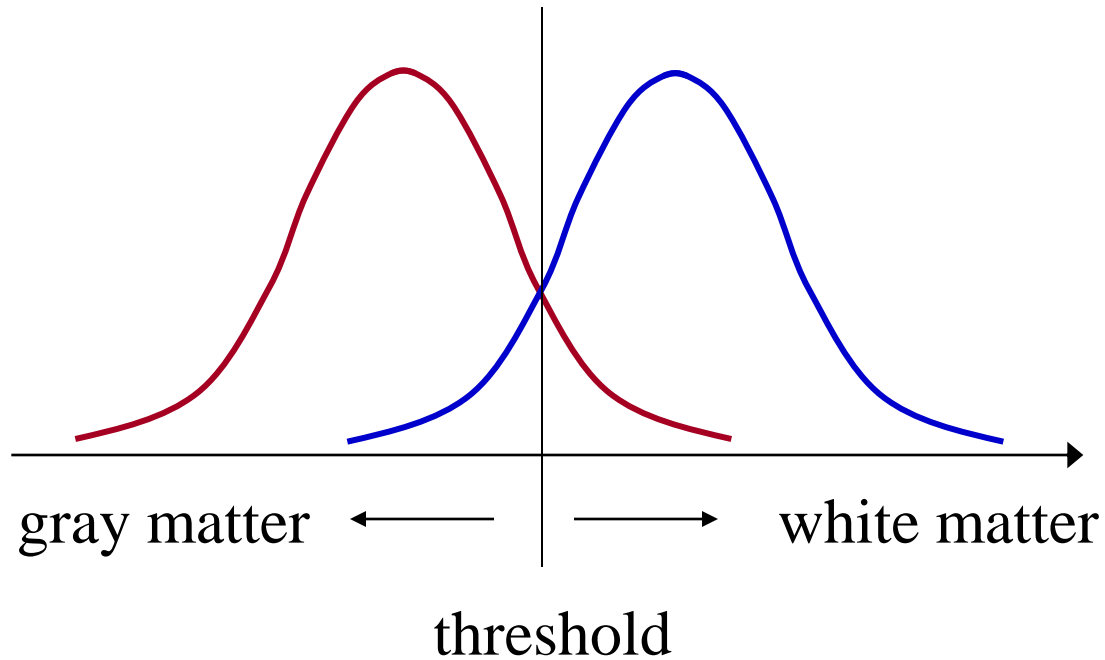
Maximum Likelihood Classification

- Measure intensity, I_0 , and we want to know the tissue class

$$L(TC_j) = p(I_0 | TC_j)$$

- Pick tissue class that maximizes L
- L is not a probability
 - Called: Likelihood

Example - revisited



Anatomical Knowledge

- *A priori* model
 - Before the measurement is considered

$$P(TC_j)$$

MAP Classifier

- Choose TC to Maximize the *A Posteriori* probability

The diagram shows the equation for the Maximum A Posteriori (MAP) classifier, $P(TC | I_0) = \frac{p(I_0 | TC)P(TC)}{p(I_0)}$, enclosed in a blue-bordered box. Annotations with arrows point to various parts of the equation: 'measurement' points to $p(I_0 | TC)$, 'model' points to $P(TC)$, 'prior' points to $P(TC)$, 'posterior probability' points to the entire left side of the equation, and 'not important' points to the denominator $p(I_0)$.

$$P(TC | I_0) = \frac{p(I_0 | TC)P(TC)}{p(I_0)}$$

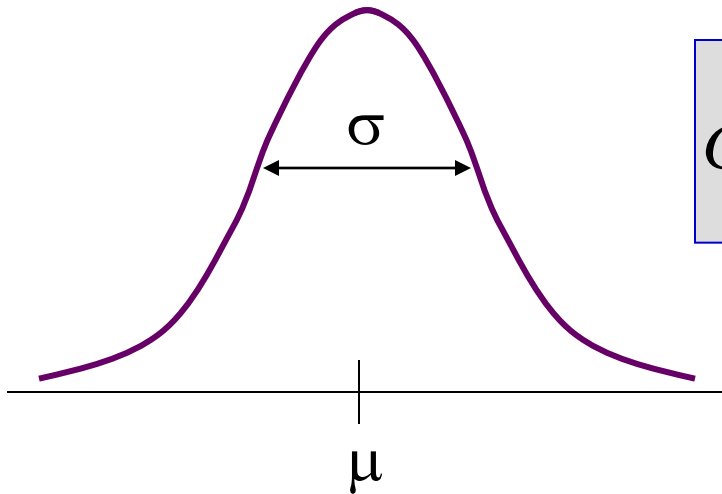
measurement
model
prior
posterior probability
not important

Measurement Model

- Training data
 - Get an expert to label some of the voxels
- Optional: Use a parametric model
 - Assume functional form
 - Popular choice: Gaussian

Gaussian Density – 1D

- Why?
 - Central Limit Theorem
 - Makes math easy (when doing parameter estimation)



$$G(\mu, \sigma, x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Choosing σ and μ

- Use training data: $\{I_1, I_2, \dots, I_N\}$
- ML parameter estimation

$$\mu = \frac{1}{N} \sum_i I_i$$

$$\sigma^2 = \frac{1}{N} \sum_i (I_i - \mu)^2$$

- MAP tissue classifier with Gaussian measurement model: choose tissue class to maximize:

$$P(TC_j | I) = \frac{G(\mu_j, \sigma_j, I)P(TC_j)}{\dots}$$

Gaussian Density – 2d Data

- Example

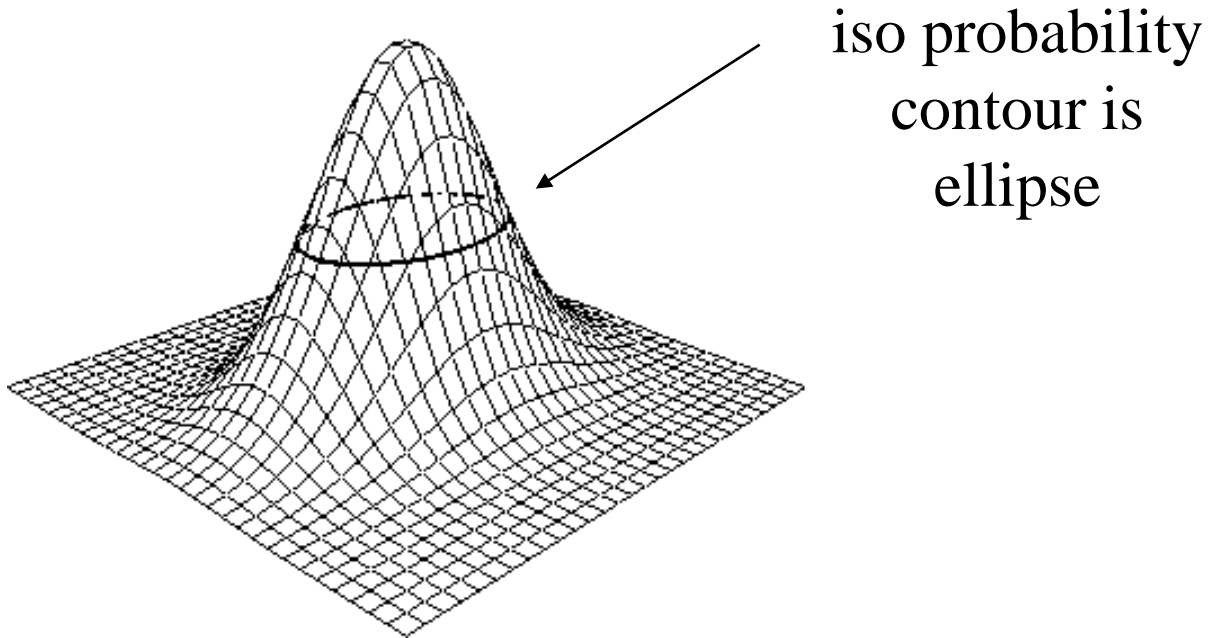
$$I = \begin{pmatrix} \text{proton density intensity} \\ \text{T2 weighted intensity} \end{pmatrix}$$

Mean Vector M

Covariance Matrix Σ

$$G(M, \Sigma, X) = \frac{1}{(2\pi)^{\frac{N}{2}} |\Sigma|^{\frac{1}{2}}} e^{-\frac{1}{2}(I-M)^T \Sigma^{-1} (I-M)}$$

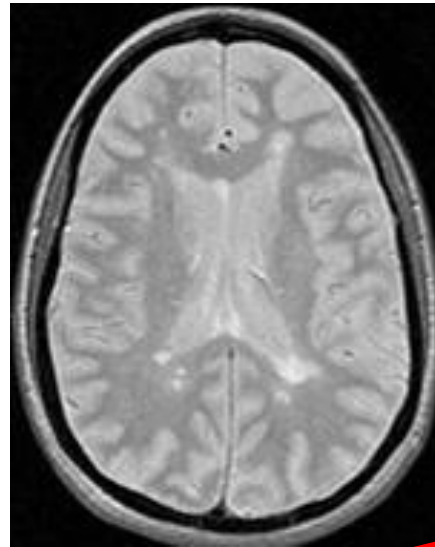
2D Gaussian: Example



Multiple Sclerosis Example

- Dual echo MRI
 - 1 x 1 x 3 mm
 - Registered slice pairs
- Proton density image
 - Good: white/gray
 - Bad: gray/CSF
- T2-weighted image
 - Not so good: white/gray
 - Good: CSF/MS lesions

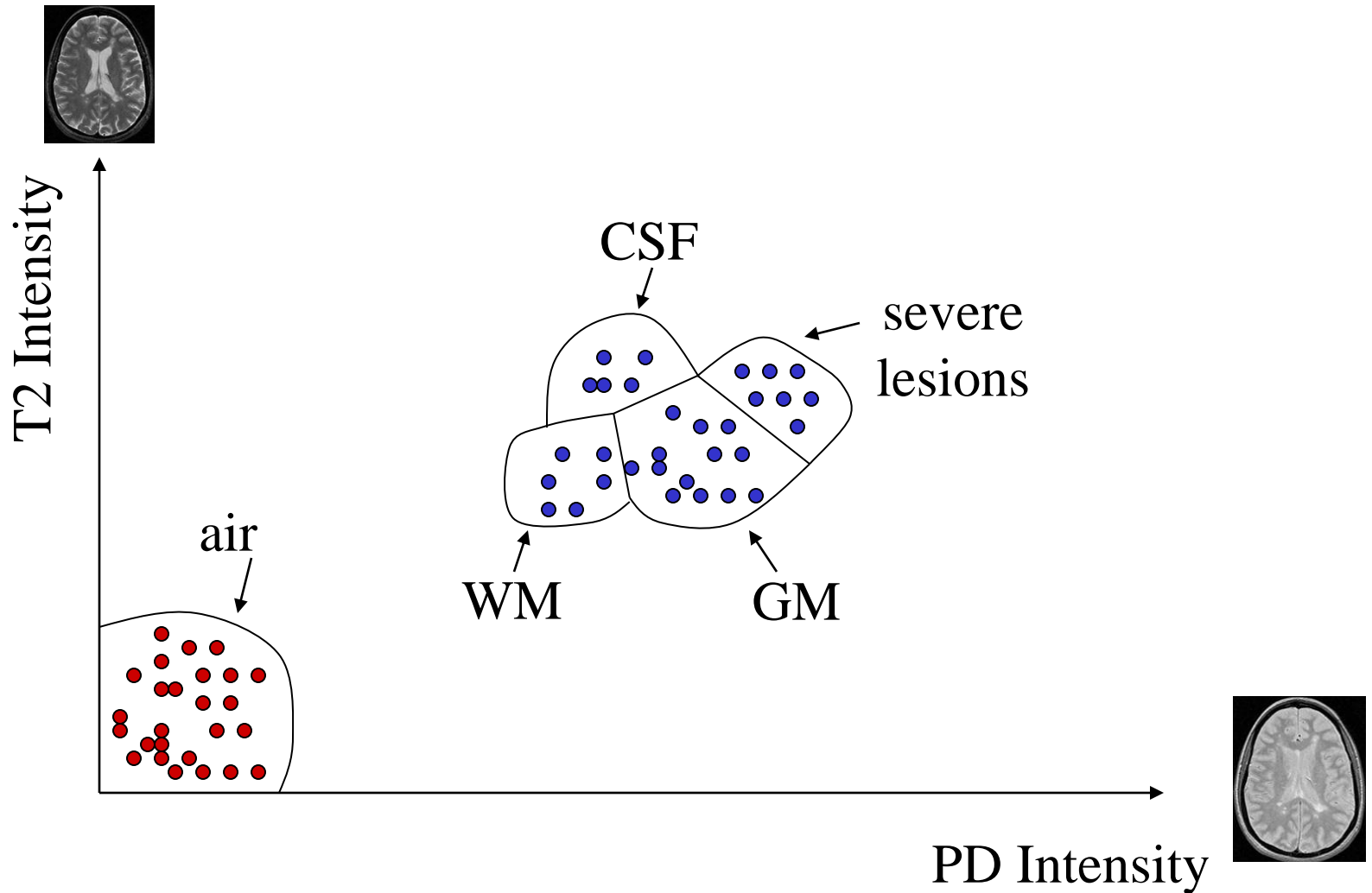
PDw



T2w

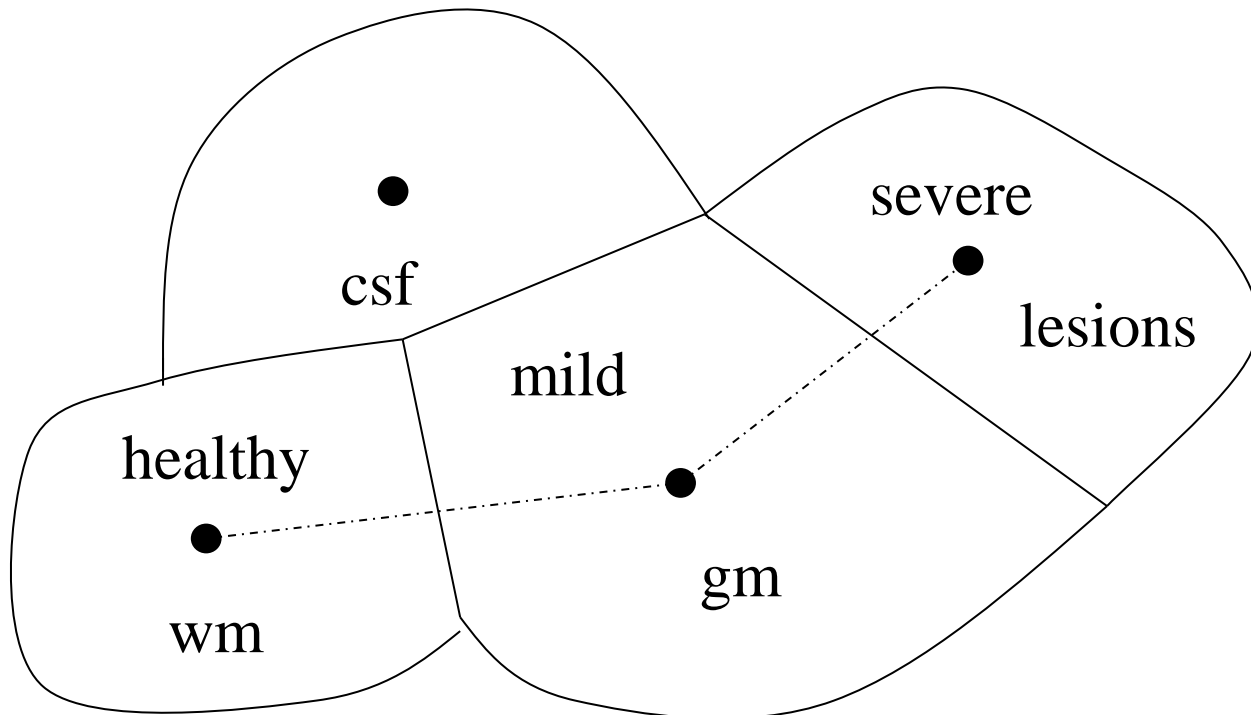


Dual Echo MRI Feature Space

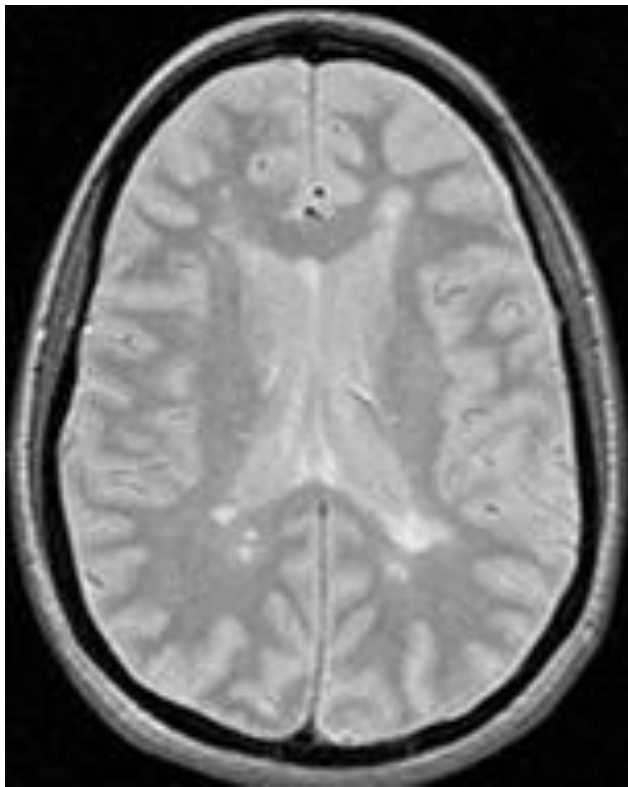


Detail

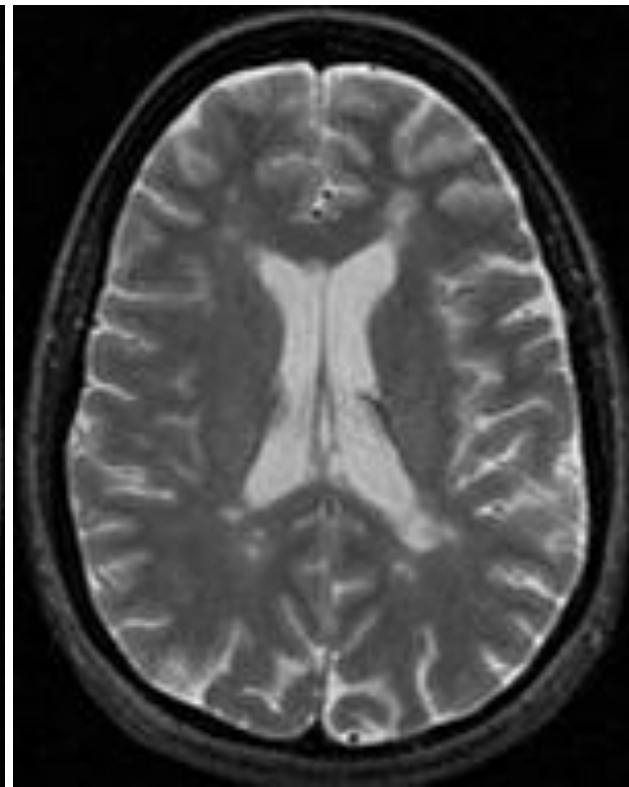
- MS Lesions are “graded phenomenon” in MRI, and can be anywhere on the curve



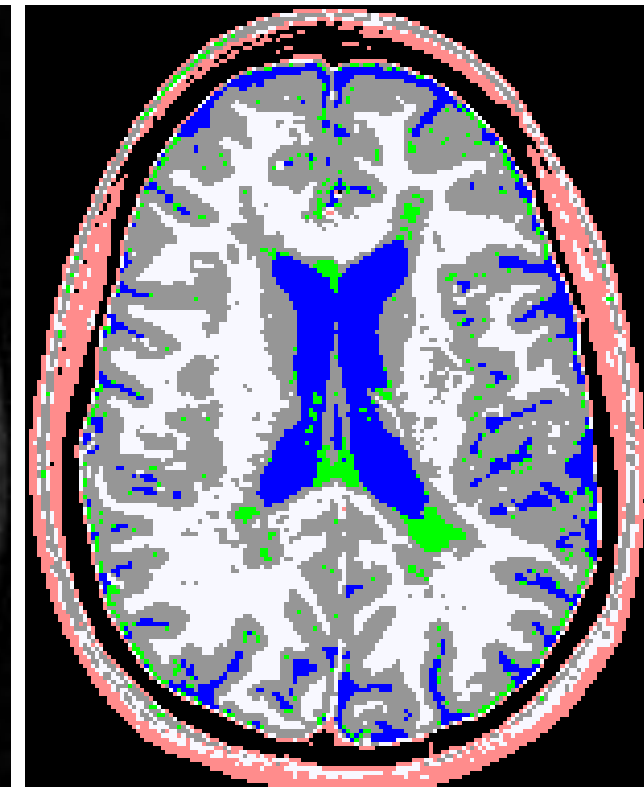
Multiple Sclerosis



PDw



T2w



Segmentation

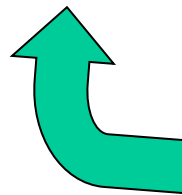
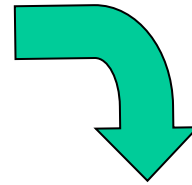
Background: Intensity Inhomogeneities in MRI

- MRI signal derived from RF signals...
- Intra Scan Inhomogeneities
 - “Shading” ... from coil imperfections
 - interaction with tissue?
- Inter Scan Inhomogeneities
 - Auto Tune
 - Equipment Upgrades

EM-Segmentation

E-Step

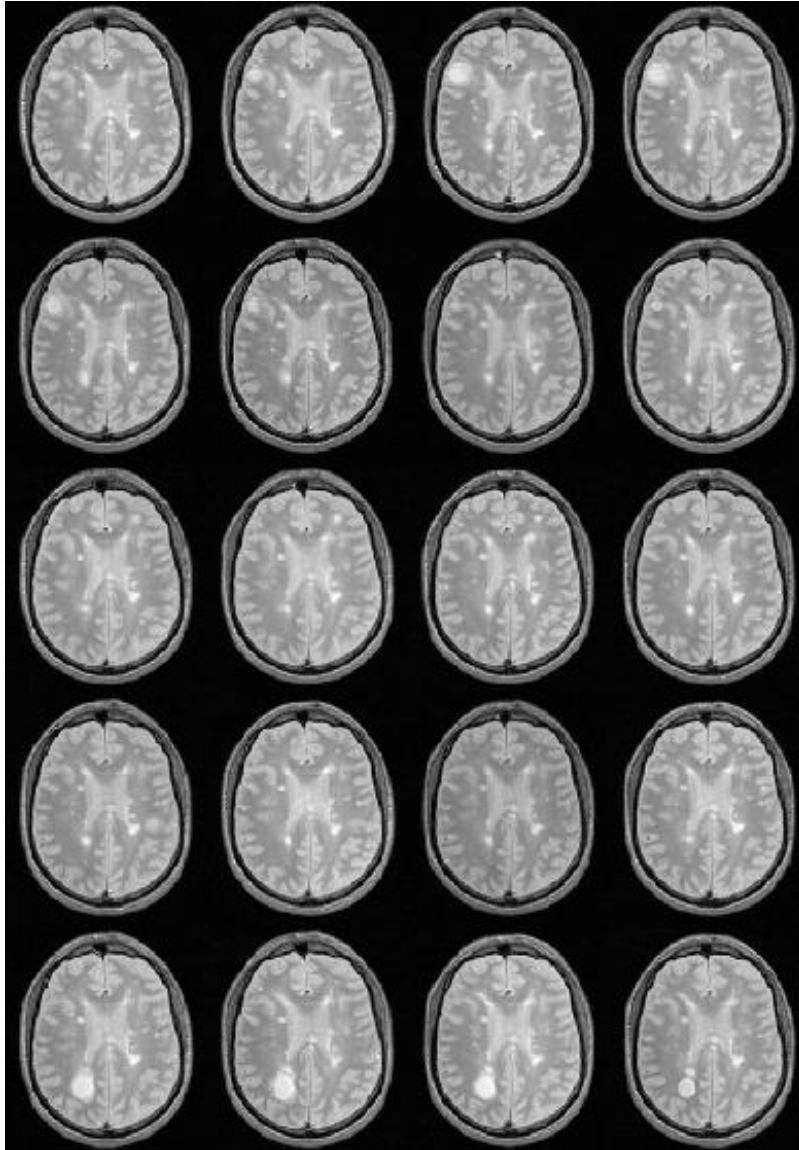
Compute tissue posteriors using current intensity correction.



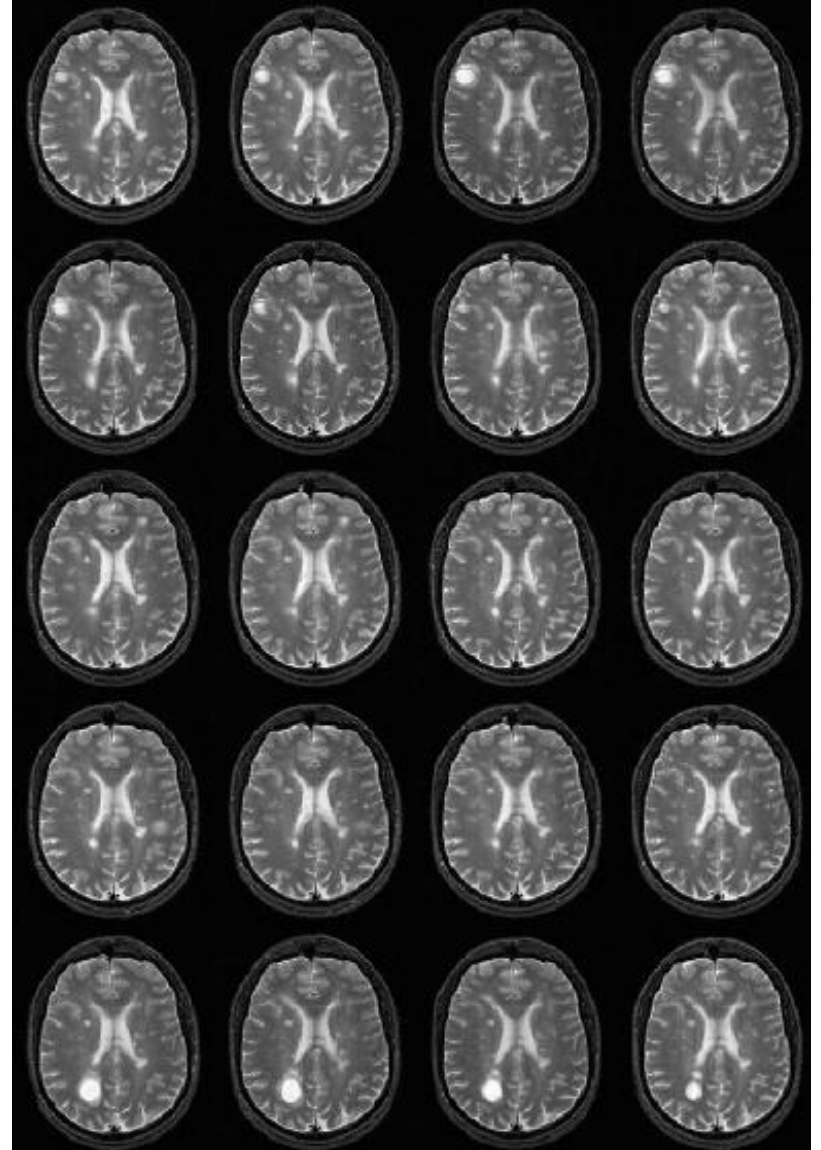
Estimate intensity correction using residuals based on current posteriors.

M-Step

Dual Echo Longitudinal Study

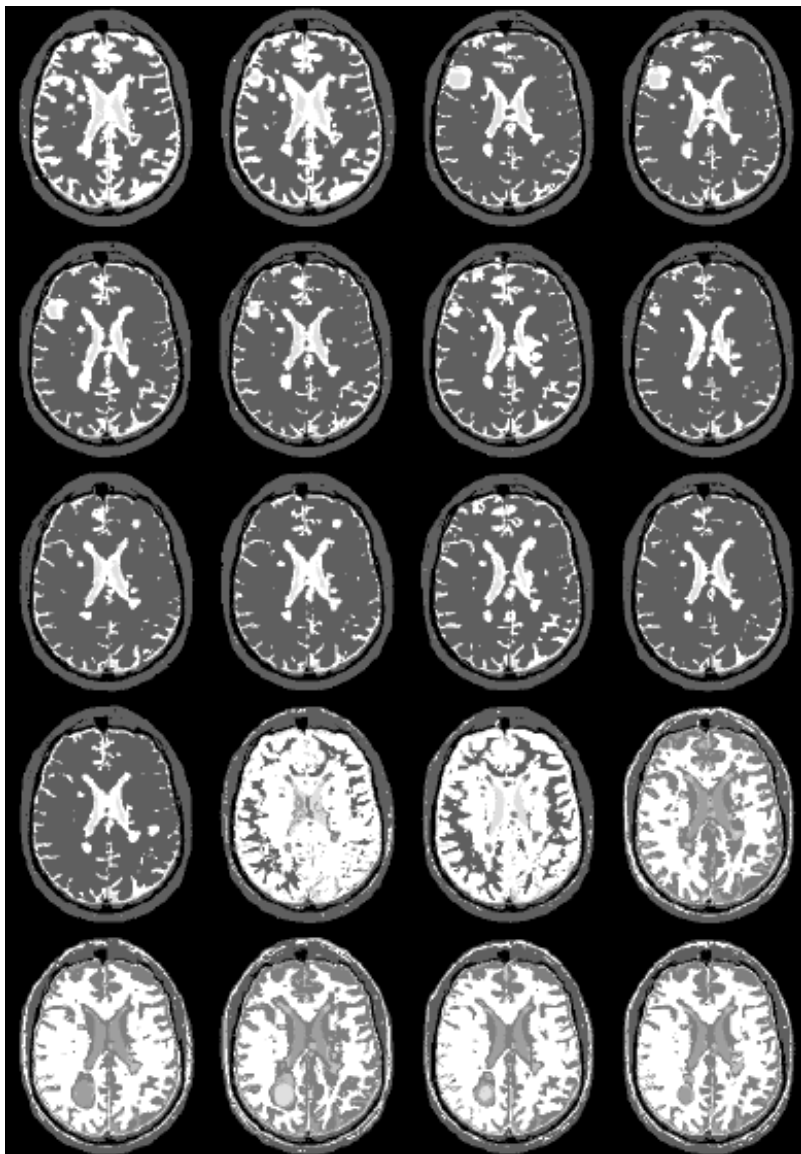


PDw

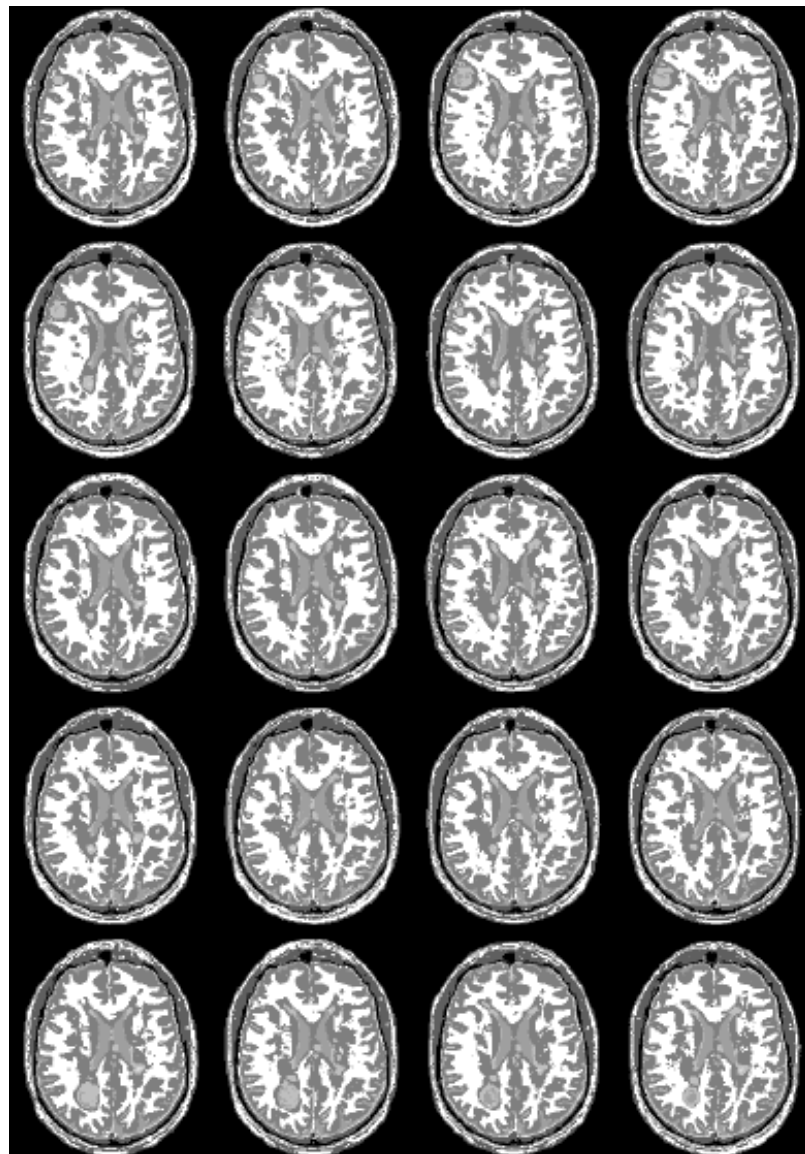


T2w

Tissue classification

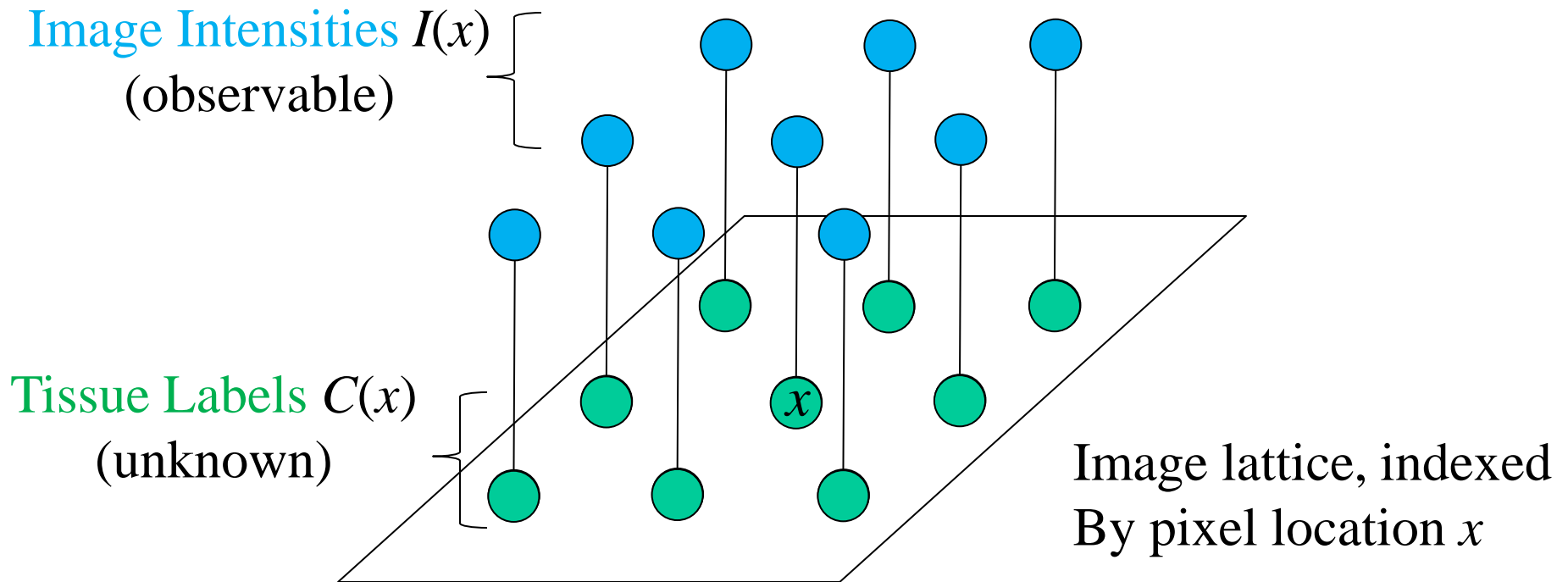


No Intensity Correction



EM Segmentation

Generative Model

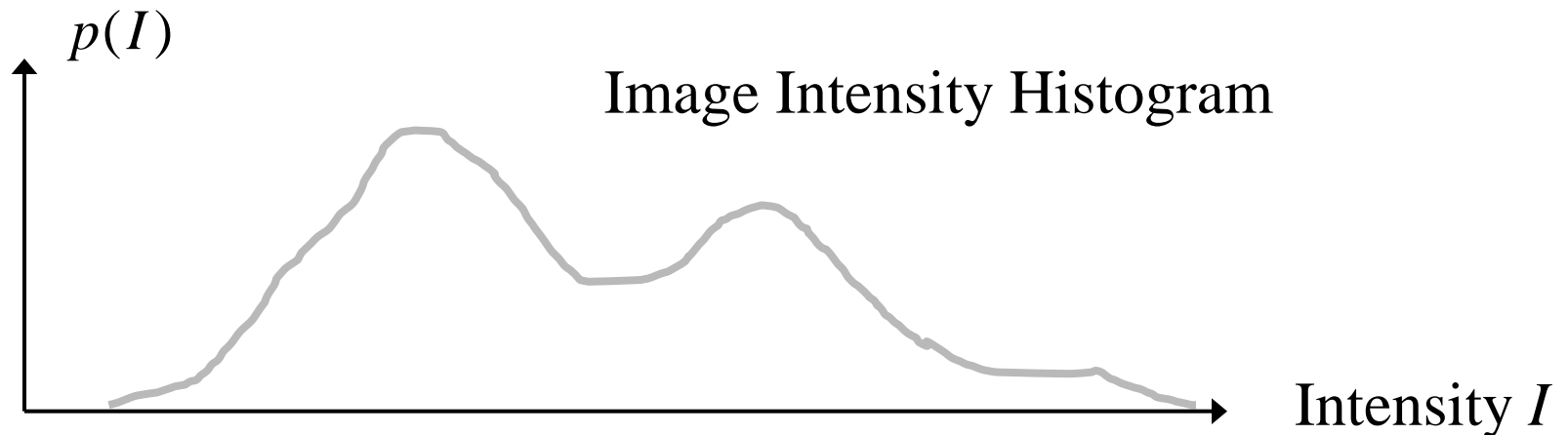


Gaussian Mixture Model (GMM)

Likelihood
(e.g. Gaussian density)

Prior
(discrete)

$$p(I) = \sum_i p(I | C_i) P(C_i)$$



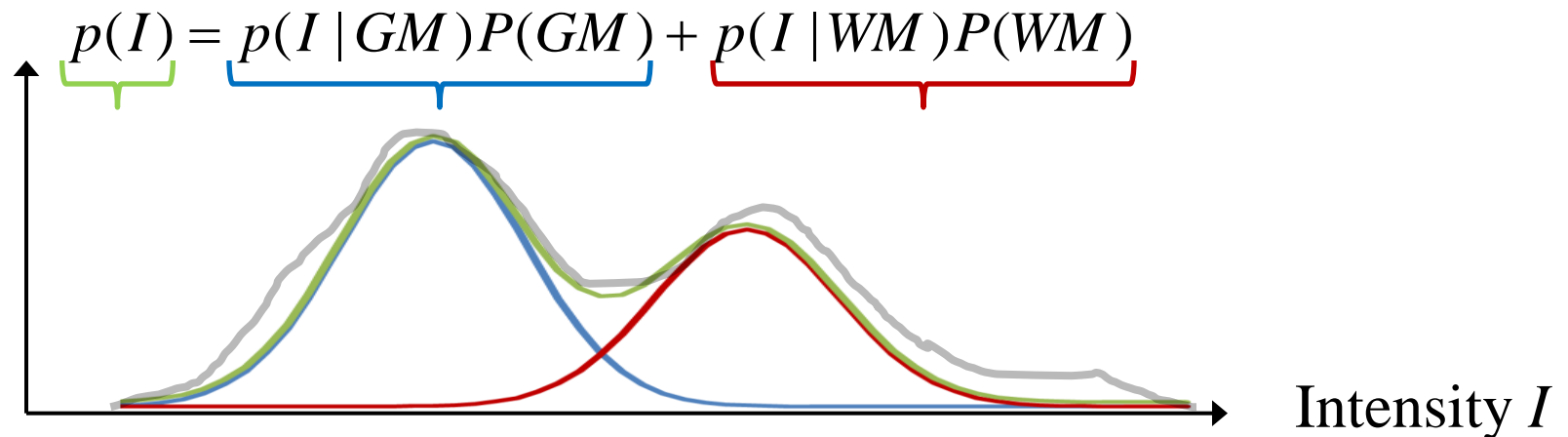
GMM Example

Two Tissue Types

Grey Matter (GM), White Matter (WM)

ML Parameter Estimation

- Expectation-Maximization (EM) Algorithm



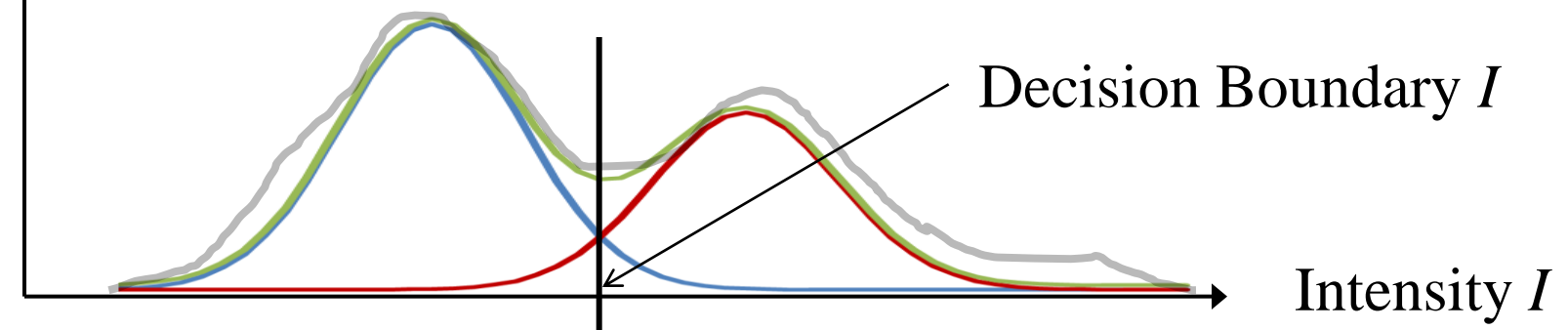
GMM Segmentation

Maximum A-Posteriori (MAP) Tissue Estimation

- Identify most probable C given intensity sample I

$$p(C_i | I) \propto p(I | C_i)P(C_i)$$

$$p(I) = \underbrace{p(I | GM)P(GM)}_{\text{blue}} + \underbrace{p(I | WM)P(WM)}_{\text{red}}$$

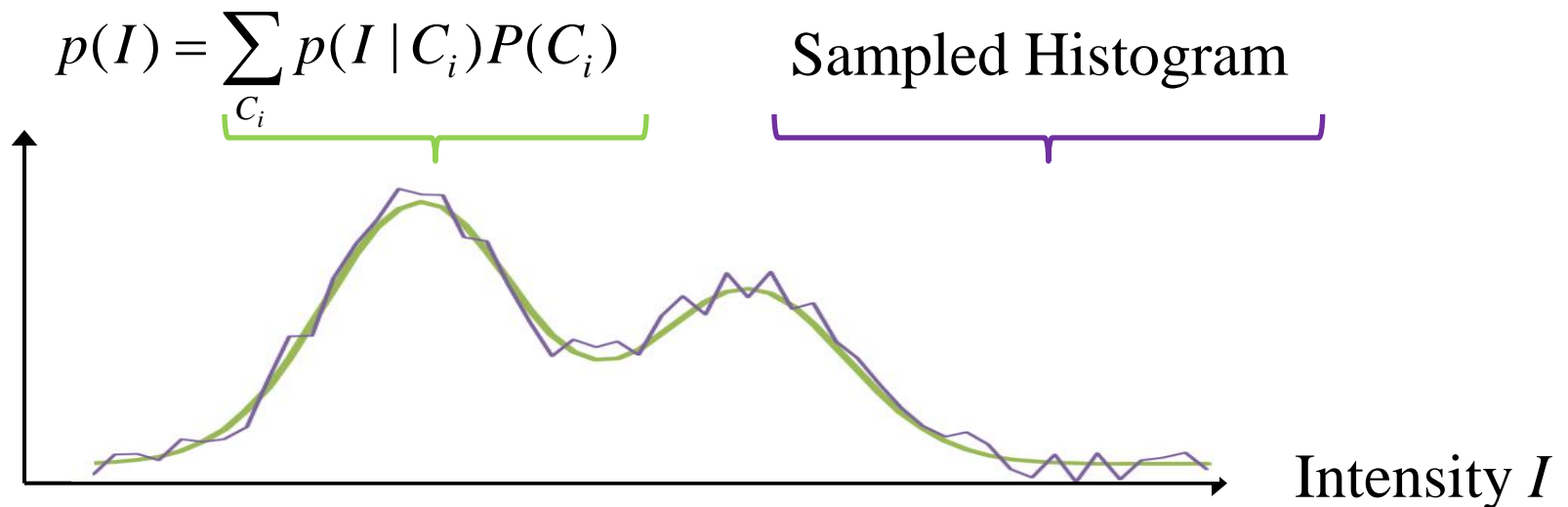


GMM Sampling

Generate new image histogram

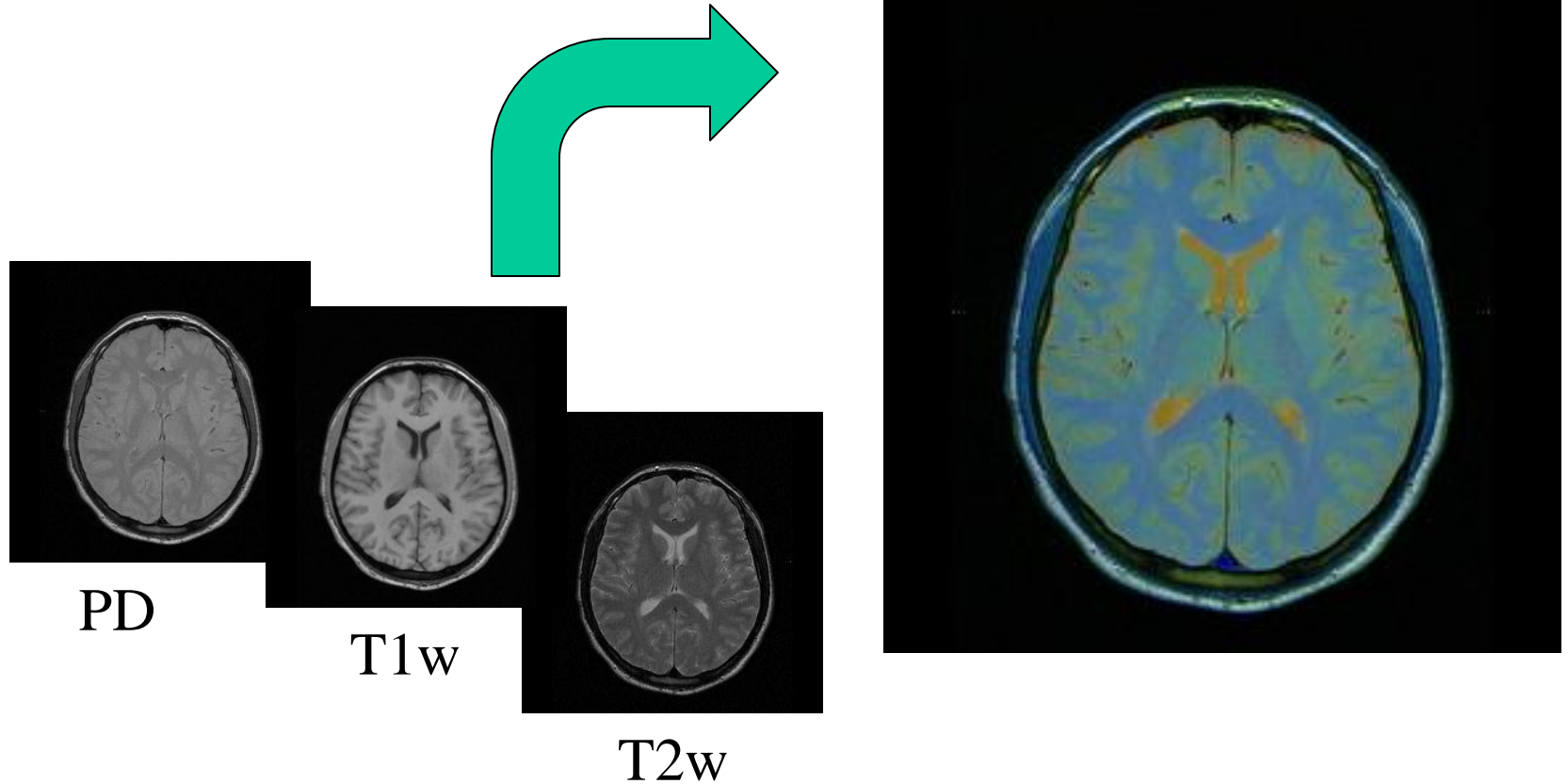
- draw samples from GMM

Check modeling assumptions



GMM Image Example

Visualize as RGB Colors

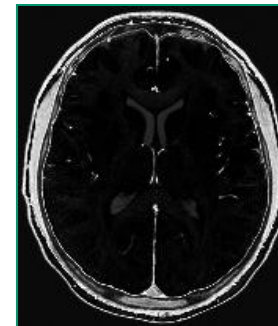
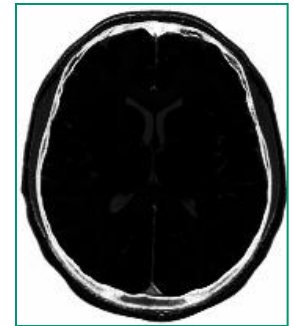
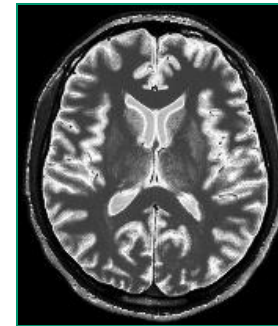
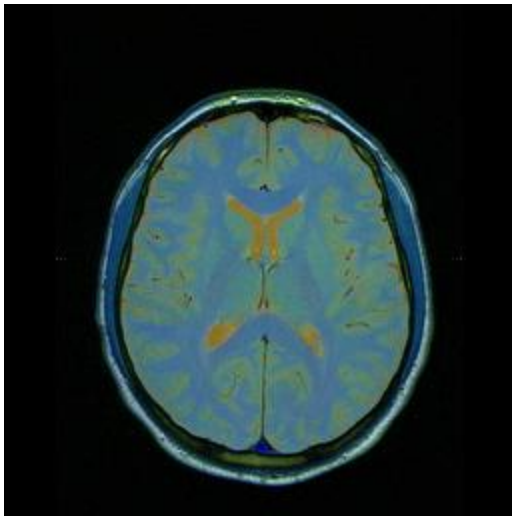


GMM Image Example

$$p(I) = \sum_i p(I | C_i) P(C_i)$$

Learn 4-tissue model

$$p(C_i | I) \propto p(I | C_i) p(C_i)$$

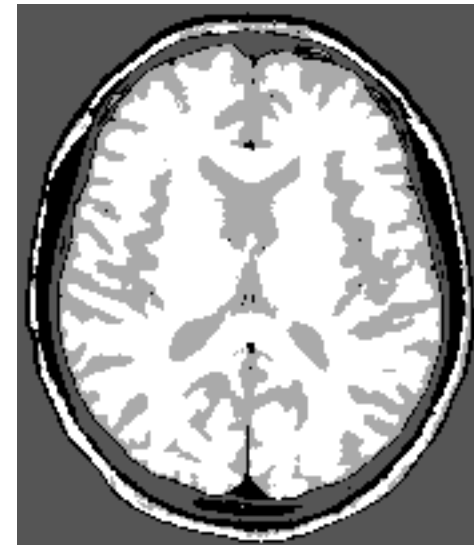
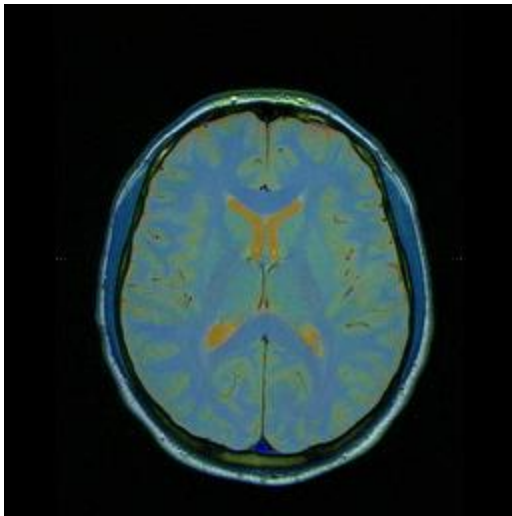


GMM Image Example

$$p(I) = \sum_i p(I | C_i) P(C_i)$$

MAP Tissue Class Estimation

$$p(C_i | I) \propto p(I | C_i) p(C_i)$$

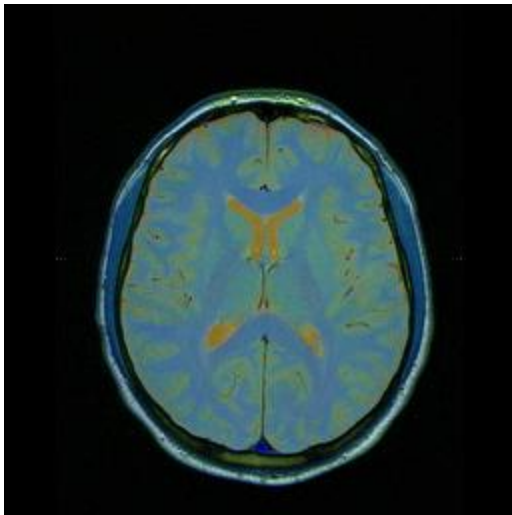


GMM Image Example

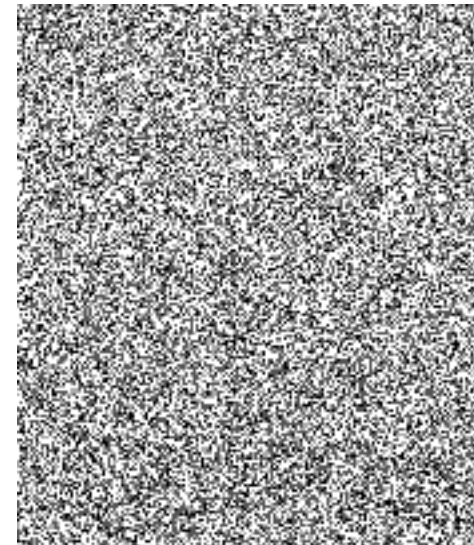
$$p(I) = \sum_i p(I | C_i) P(C_i)$$

Generate new image from model

$$p(C_i | I) \propto p(I | C_i) p(C_i)$$



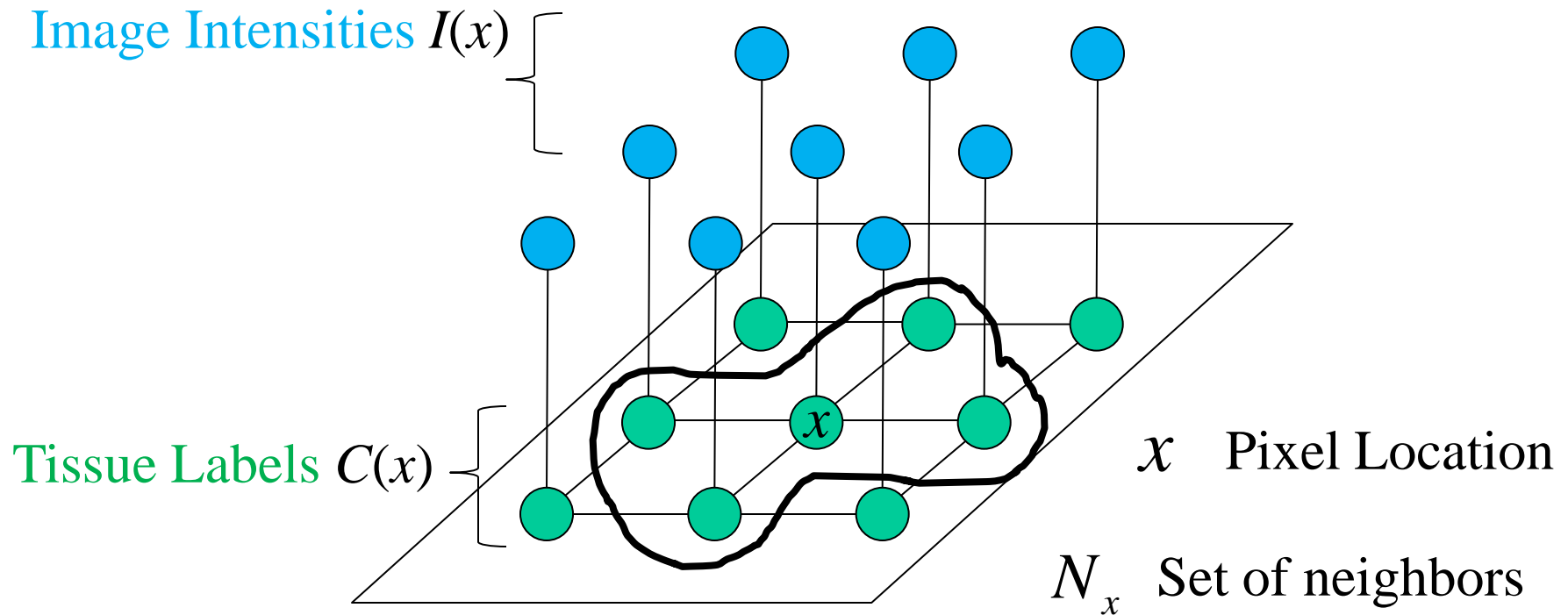
No spatial
component



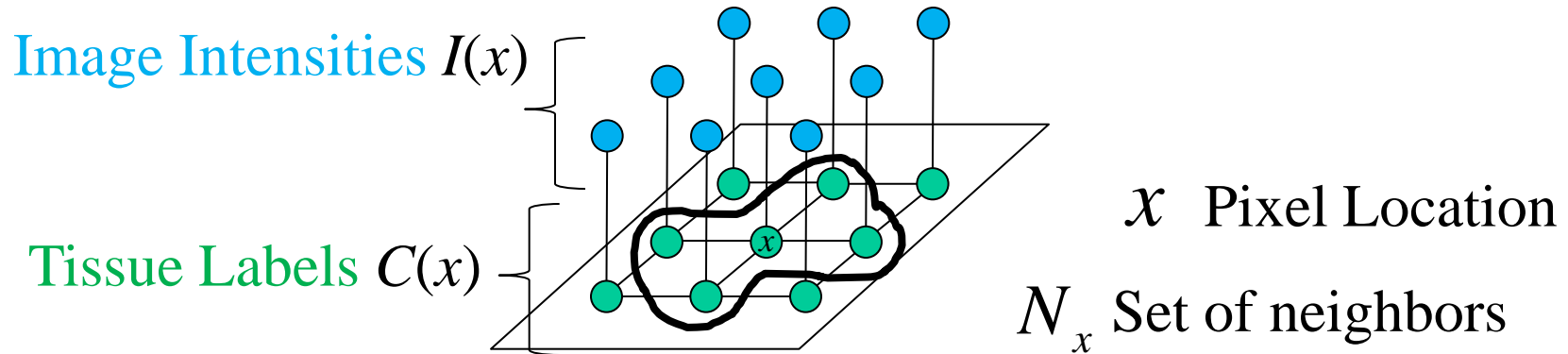
Prior Models

- Markov Random Fields (MRF)
- Structurally-Conditioned Models
- Average Brain

Markov Random Fields (MRF)



Markov Random Fields (MRF)



Markov Assumption

Label at x independent of all others, given N_x

Hammersley–Clifford Theorem

Probability is a Gibbs distribution over all neighborhood ‘cliques’ $Cl(X)$

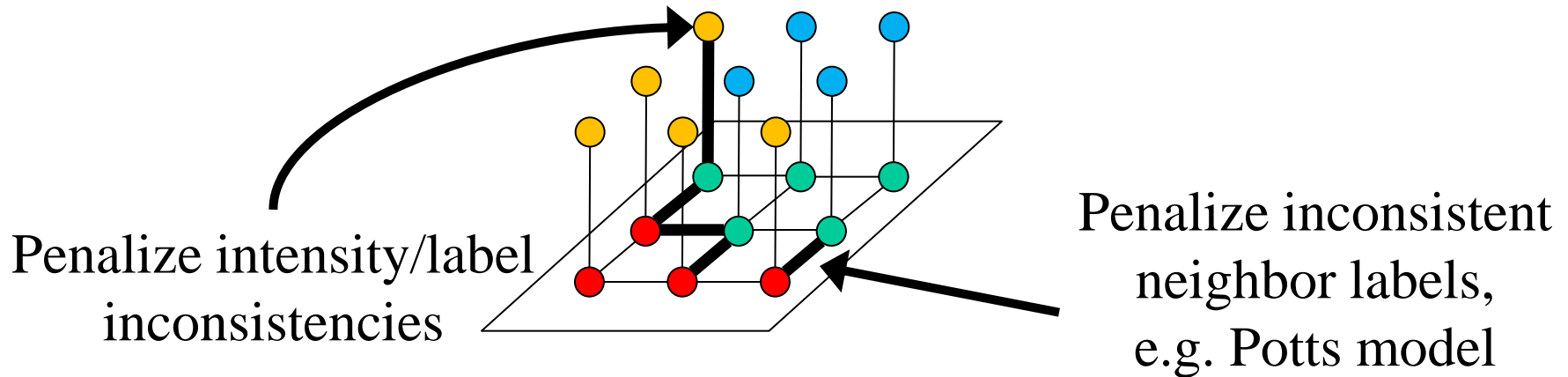
Energy Function:

Positive



$$p(C | I, X) \propto \frac{1}{Z} \exp \sum_{\xi \in Cl(X)} -E(\xi, I, C)$$

Markov Random Fields (MRF)



$$E_1(x, I, C)$$

$$E_2(x, x', C) = \begin{cases} 1, & \text{if } C(x) = C(x') \\ 0, & \text{otherwise} \end{cases}$$

$$p(C | I, X) \propto \exp \left[- \sum_{x \in X} E_1(x, I, C) - \sum_{x' \in N_x} E_2(x, x', C) \right]$$

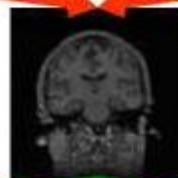
Average Brain Models

- Construct a spatial prior model by averaging tissue distributions over a population [MNI].

Training Subjects



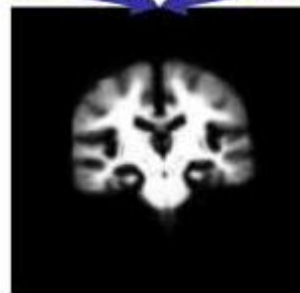
Generic Subjects



Segmentations



Spatial Prior

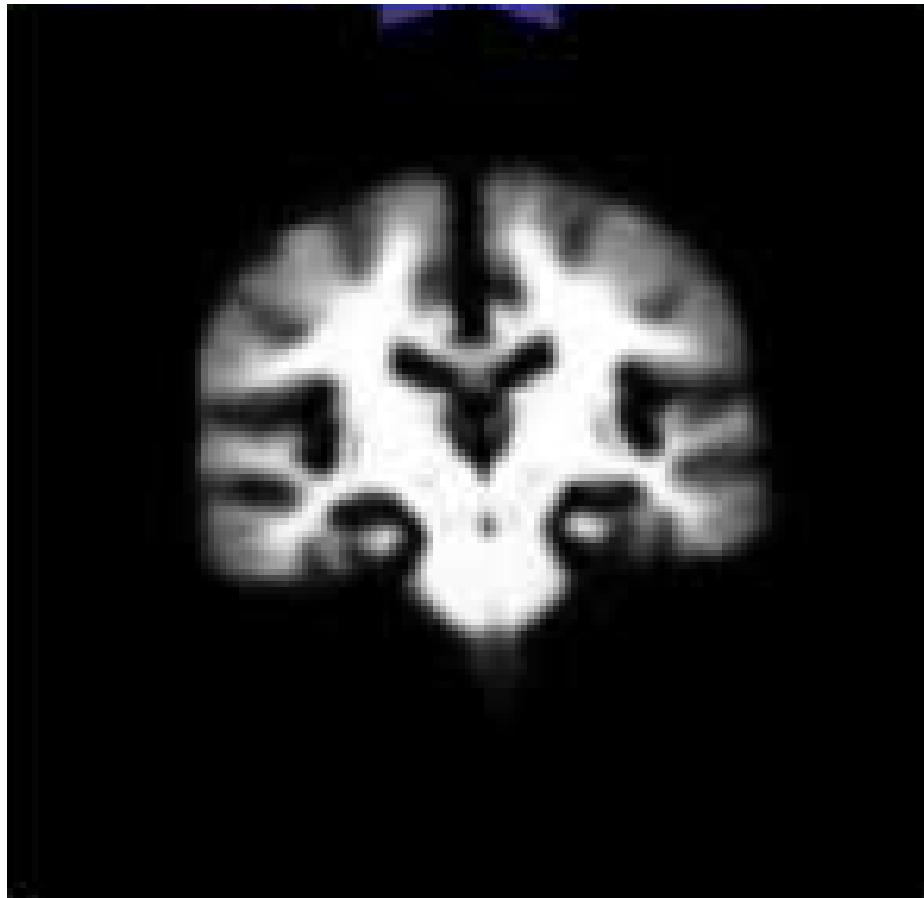


→ register MRIs

→ align Segmentation

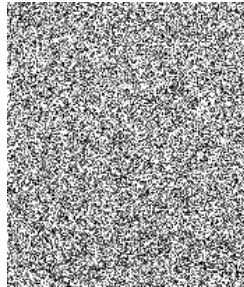
→ produce prior

$P(\text{white matter} \mid x)$

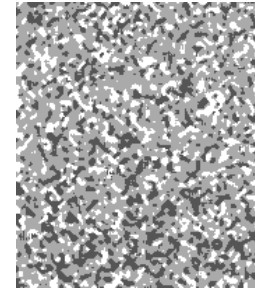


Generated Sample Images


GMM: No spatial component

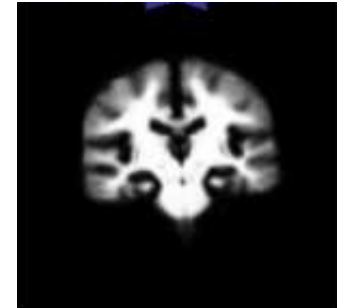


MRF: Spatial neighborhood



Average Brain: GMM conditioned on location x

$$p(I | x) = \sum_i p(I | C_i) P(C_i | x)$$




Note conditional independence assumption:

$$p(I | C_i) = p(I | C_i, x)$$

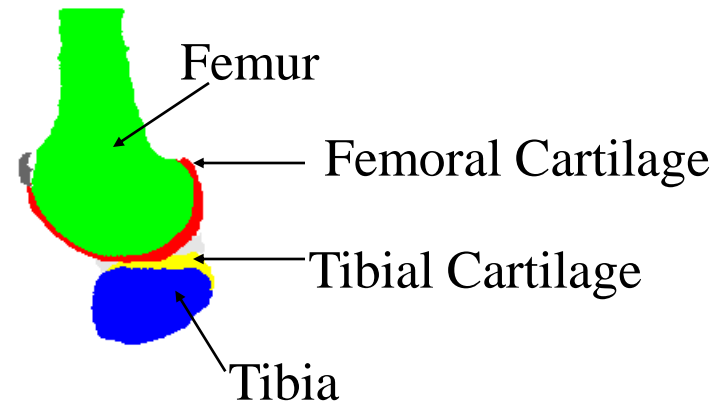
Structurally-Conditioned Prior Models

- From (Kapur 1999)
 - Modeling Global Geometric Relationships
between Structures

Modeling Global Geometric Relationships between Structures

- Relative Geometry Models
- Motivate Using Knee MRI

Segmented Knee MRI



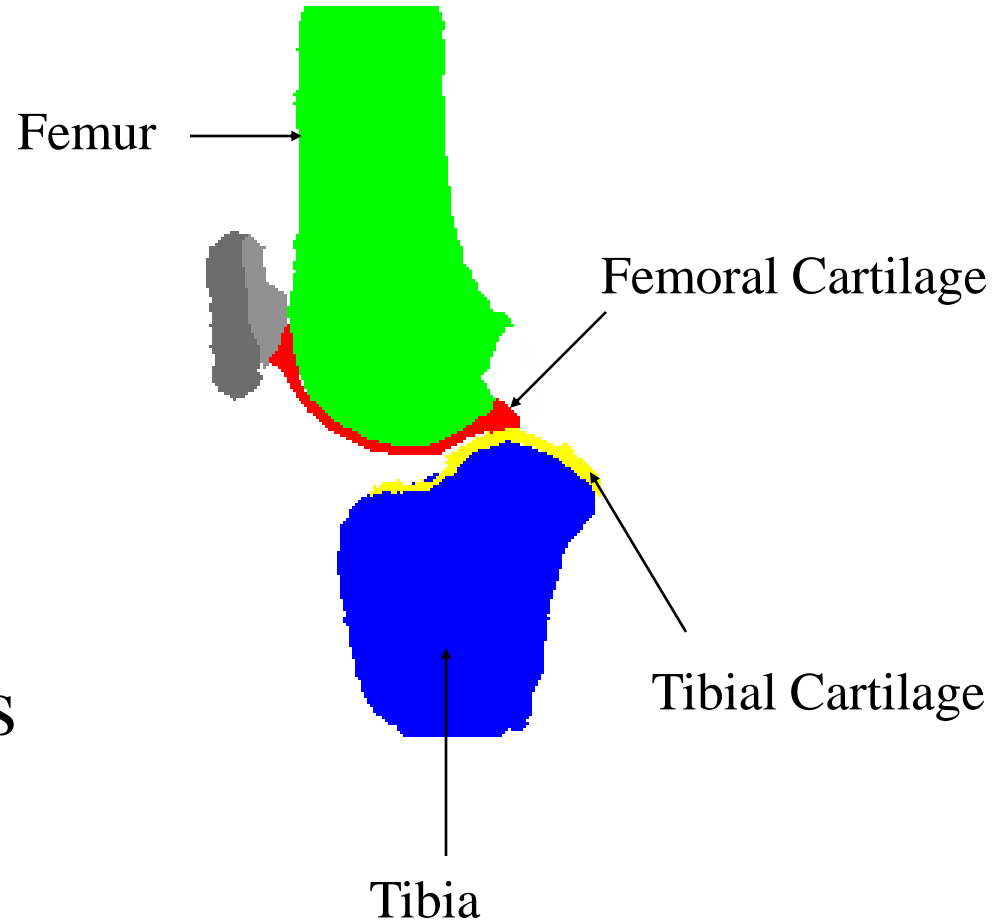
Motivation

- **Primary Structures**

- image well
- easy to segment

- **Secondary Structures**

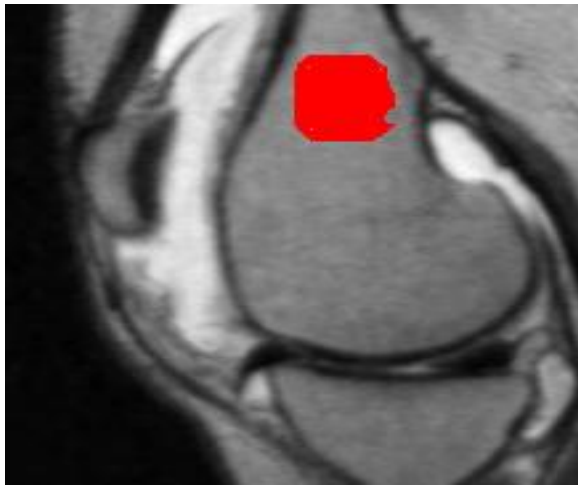
- image poorly
- relative to primary



Relative Geometric Prior Approach

- Select primary/secondary structures
- Measure geometric relation between primary and secondary structures from training data
- Given novel image
 - segment primary structures
 - use geometric relation as prior on secondary structure in EM-MF Segmentation

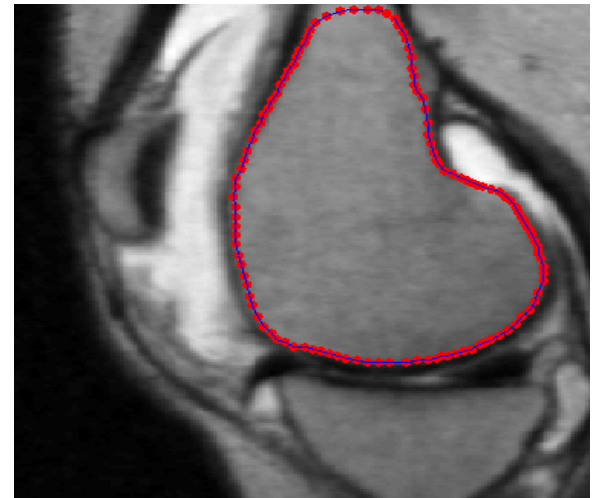
Segment Primary Structures: Femur, Tibia



Seed



Region Growing

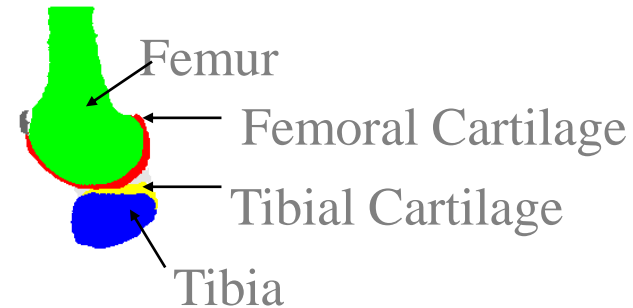


Boundary Localization

Status

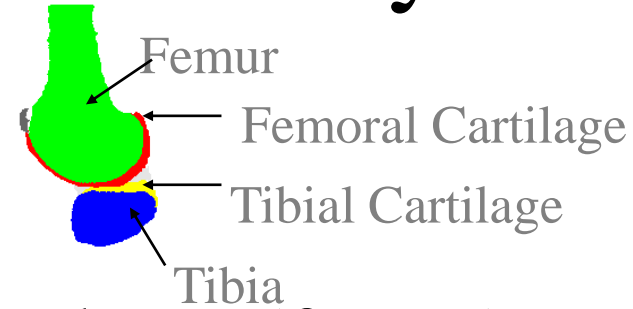
- Have Bone
- Want Cartilage

Measure Geometric Relationship between Primary and Secondary Structures



- Using primitives such as
 - distances between surfaces
 - local normals of primary structures
 - local curvature of primary structures
 - etc.

Measure Geometric Relationship between Primary and Secondary Structures



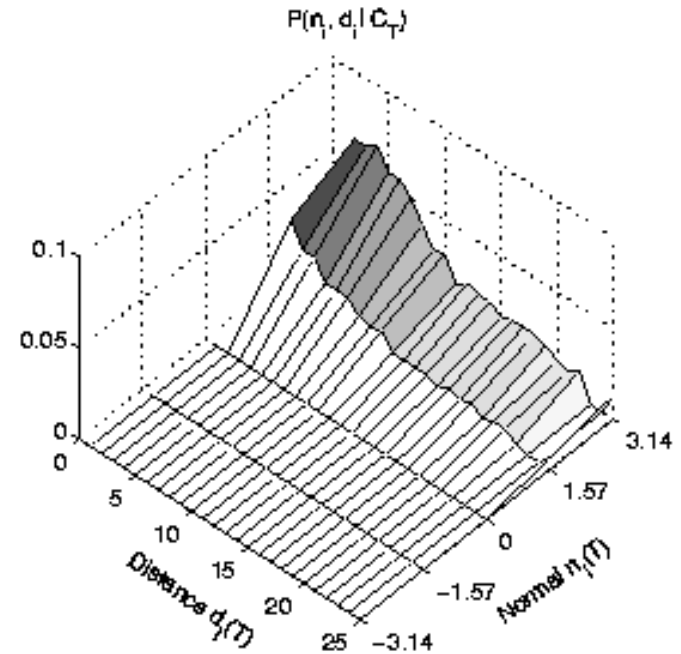
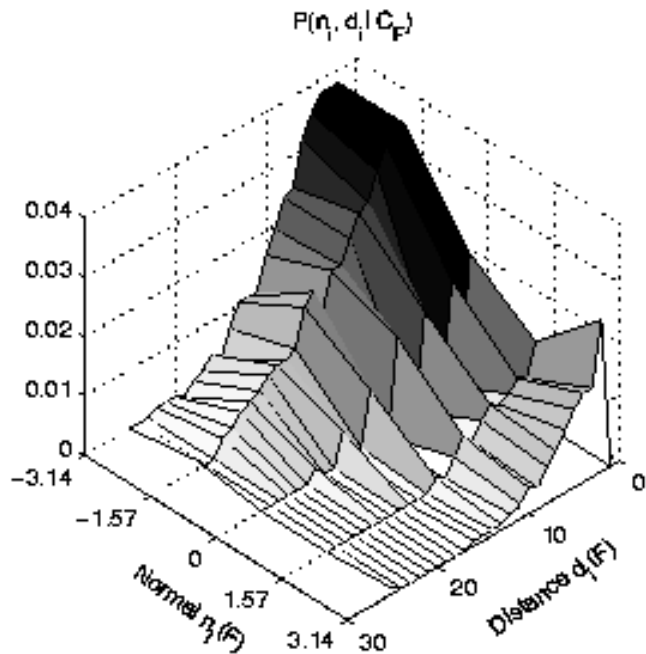
$\rho_s \equiv$ distance to closest point on bone (femur)

$n_s \equiv$ normal to bone (femur) at closest point

$$P(x_s \in \text{Cartilage} \mid \text{Bone})$$

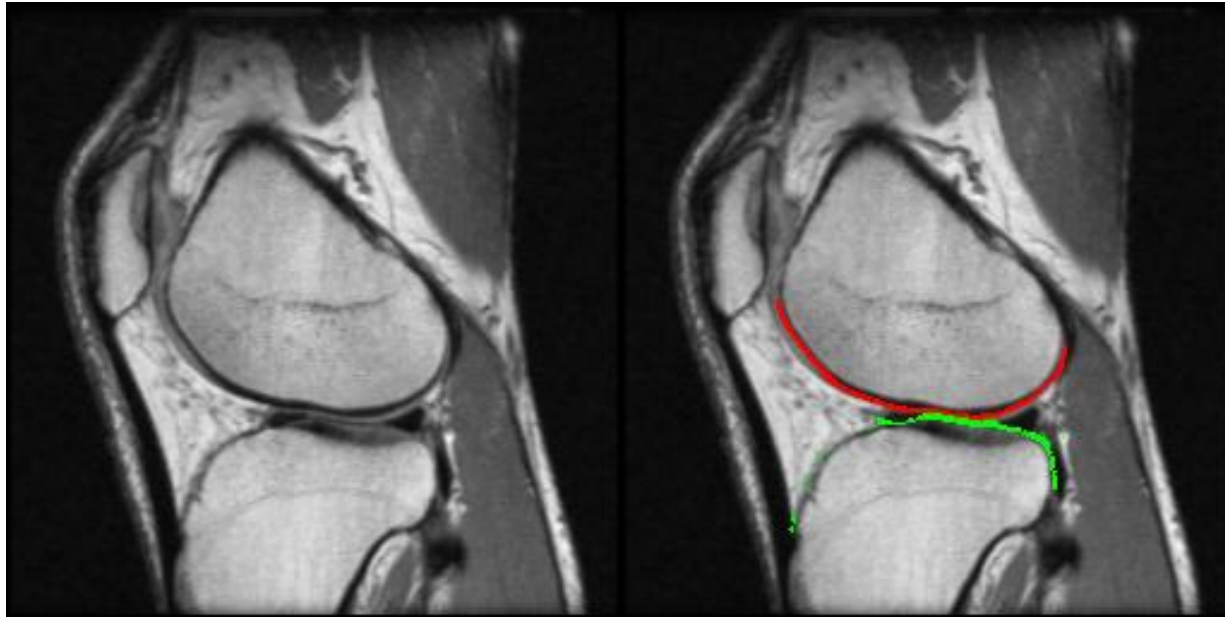
$$\approx \frac{P(\rho_s, n_s \mid x_s \in \text{Cartilage})P(\text{Cartilage})}{Z}$$

Estimate of $P(\rho_s, n_s | x_s \in \text{Cartilage})$



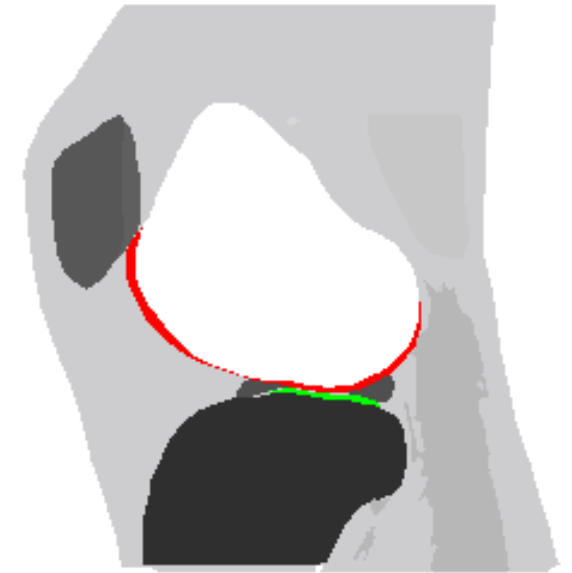
$P(\rho_s, n_s | x_s \in \text{Fem. Cartilage})$ $P(\rho_s, n_s | x_s \in \text{Tib. Cartilage})$

Results: Segmentation of Femoral & Tibial Cartilage



MRI Image

Model-Based
Segmentation



Manual Segmentation

Morphological Operations

- Erosion
- Dilation
- Opening
- Closing

- [Haralick + 1989]

Morphological Operators...

- Ubiquitous simple tools. Useful for ad-hoc clean-up of results from Statistical Classification.

Dilation

- Binary (or Boolean) images
- Represent image by a set of coordinate vectors of pixels with value 1

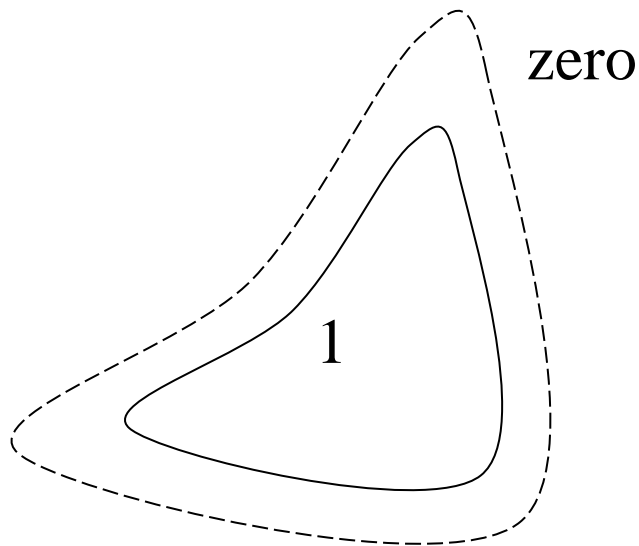
image $\rightarrow A \oplus B \equiv \{c \mid c = a + b, \text{ for some } a \in A, b \in B\}$

vector addition

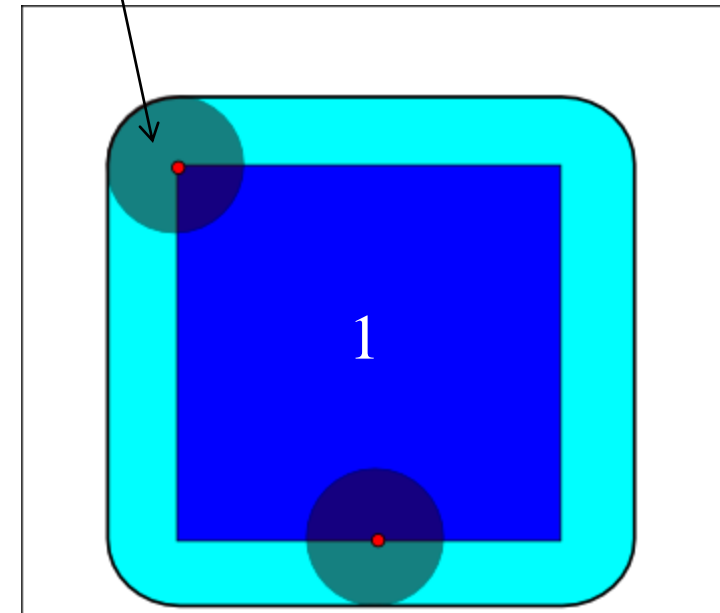
Typical structure elements: $\left\{ \begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array} \right.$ $\begin{array}{ccc} & & 1 \\ 1 & 1 & 1 \\ & & 1 \end{array}$

Dilation

- Continuous analogy
- Makes structures *fatter*



Structure element



Source: Wikipedia

Erosion

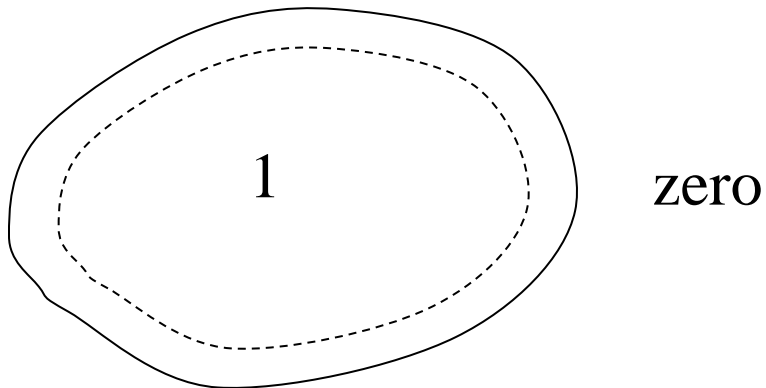
- Erosion is dual of dilation
 - complement A
 - reflect B (negate coordinates)
 - dilate
 - complement result

$$A \otimes B = \overline{\overline{A} \oplus \hat{B}}$$

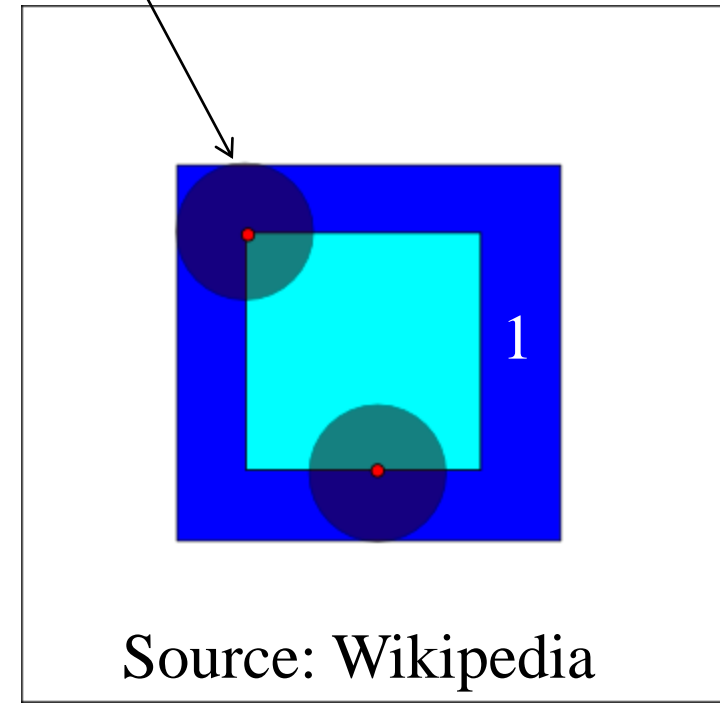
- Frequently, B is symmetric and then reflection can be ignored

Erosion

- Erosion by simple S.E.'s makes structures thinner
- Analog analogy:



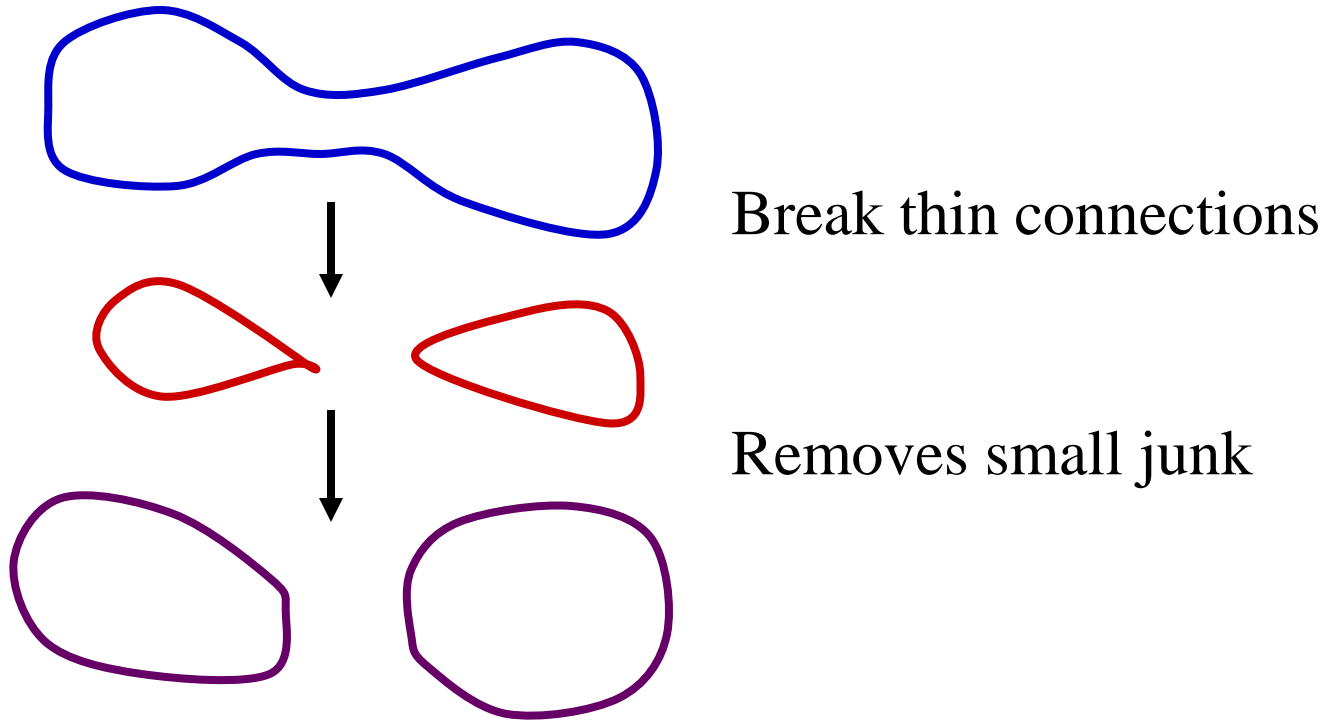
Structure element



Source: Wikipedia

Opening

- Opening = Erode then Dilate

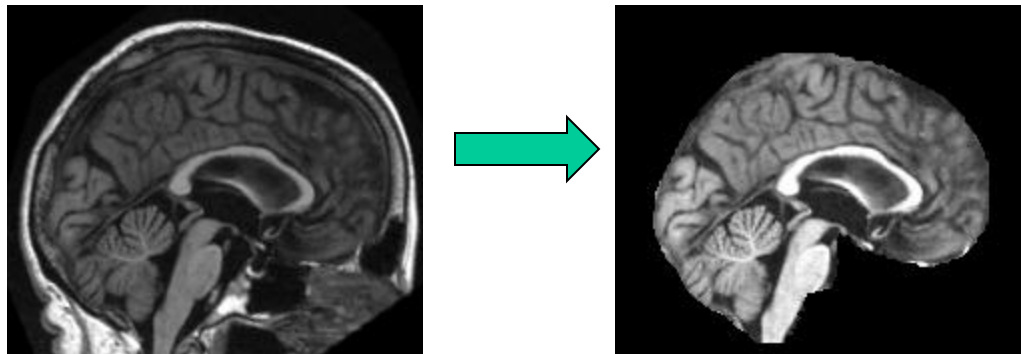


Closing

- Closing = Dilate then Erode
- Can attach objects that have become fragmented

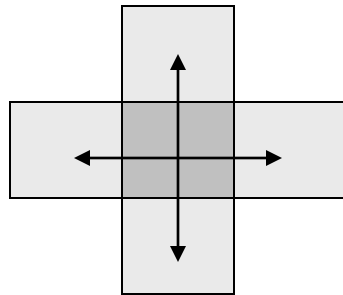
Erosion and Dilation

- Common trick in brain isolation “de-scalping”
 - Erode “it”
 - to disconnect brain from head
 - Dilate “it”
 - But *only* mark pixels that were originally “brain”

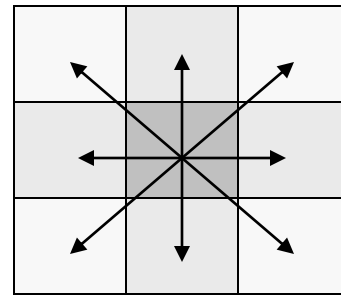


Connectivity

- Define neighbor relation



4-neighbor

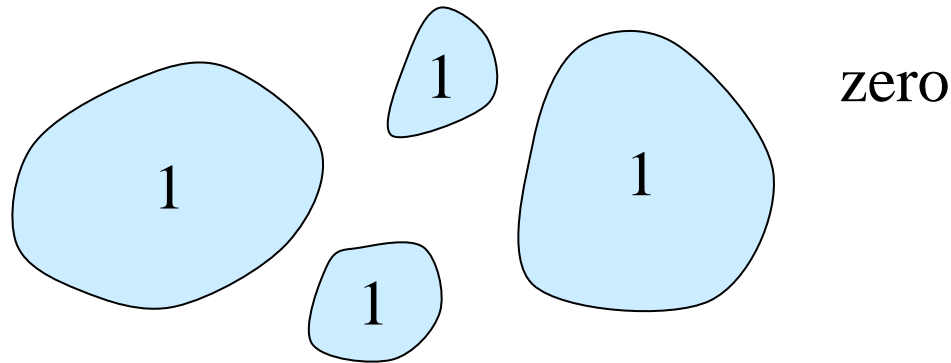


8-neighbor

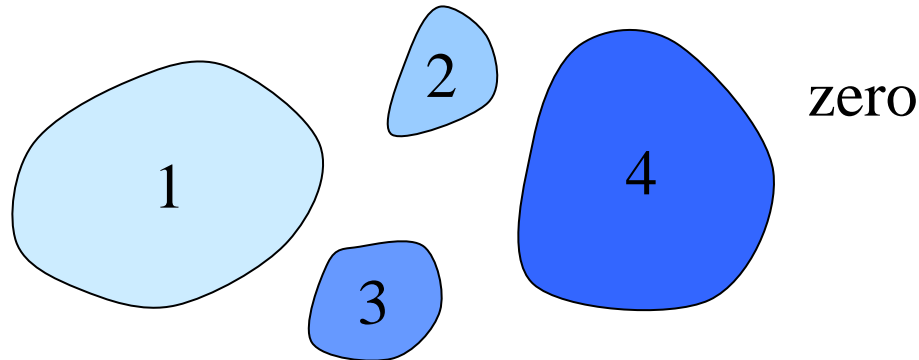
- There are some inconsistencies that a 6-neighbor relation can fix

Connected Components

- Input: Boolean image objects



- Output: Unique label for each separate object



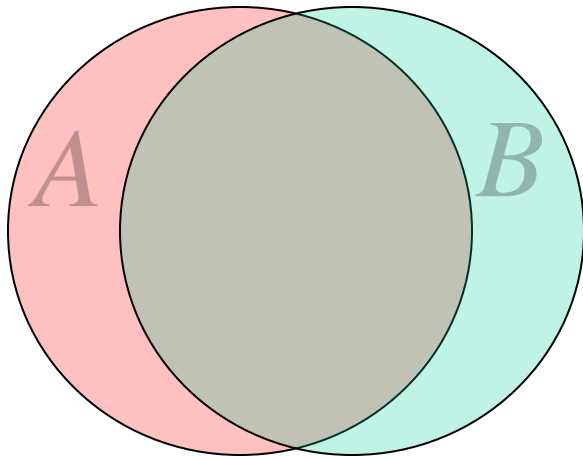
Finding Connected Components

- $N = 1$ (initialize label)
 - Repeat until all pixels are labeled
 - Pick an unmarked I pixel
 - Label it, and all of its I neighbors,
and all of their neighbors: N
 - $N \leftarrow N + 1$
- } Recursive

See Flood Fill Algorithm

Segmentation Quality Evaluation

Compare overlap between automatic and manual (ground truth) segmentation.



$$DICE \equiv 2 \frac{|A \cap B|}{|A| + |B|}$$

Dice, Jaccard Measures

Topics Not Discussed

- Edge-base segmentation
 - Active contours, level sets
- Hierarchical representations
 - Coarse-to-fine modeling

Selected References

- **[Duda and Hart1973] Duda, R. and Hart, P.1973. Pattern Classification and Scene Analysis. John Wiley and Sons.**
- **[Gonzales + 2001] R Gonzales and R Woods. Digital Image Processing, 2nd Ed. Prentice Hall 2001.**
- **[Haralick + 1989] R Haralick and S Steinberg. Image Analysis Using Mathematical Morphology. IEEE Transactions PAMI 1989.**
- **[Kapur 1999] T Kapur. Model Based Three Dimensional Medical Imaging Segmentation. PhD Thesis, MIT EECS, 1999.**
- **[Warfield + 2000] S Warfield, J Rexilius, M Kaus, F Jolesz, R Kikinis. Adaptive template moderated spatial varying statistical classification. Med. Image Analysis, 2000.**
- **[Wells + 1996] W Wells, E Grimson, R Kikinis, F Jolesz. Adaptive segmentation of MRI data. IEEE Trans. Med. Img. 15, 1996.**

Further Reading

Kass, Michael, Andrew Witkin, and Demetri Terzopoulos. "Snakes: Active contour models." International journal of computer vision 1.4 (1988): 321-331. Cited by 16117

Otsu, Nobuyuki. "A threshold selection method from gray-level histograms." Automatica 11.285-296 (1975): 23-27. Cited by 14825

Geman, Stuart, and Donald Geman. "Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images." Pattern Analysis and Machine Intelligence, IEEE Transactions on 6 (1984): 721-741. Cited by 15644

Shi, Jianbo, and Jitendra Malik. "Normalized cuts and image segmentation." Pattern Analysis and Machine Intelligence, IEEE Transactions on 22.8 (2000): 888-905. Cited by 8646

Boykov, Yuri, Olga Veksler, and Ramin Zabih. "Fast approximate energy minimization via graph cuts." Pattern Analysis and Machine Intelligence, IEEE Transactions on 23.11 (2001): 1222-1239. Cited by 4449

Zhang, Yongyue, Michael Brady, and Stephen Smith. "Segmentation of brain MR images through a hidden Markov random field model and the expectation-maximization algorithm." Medical Imaging, IEEE Transactions on 20.1 (2001): 45-57. Cited by 2201

Feature-based Analysis

- Model image as a set of image patches
- 3D scale-invariant features (SIFT)

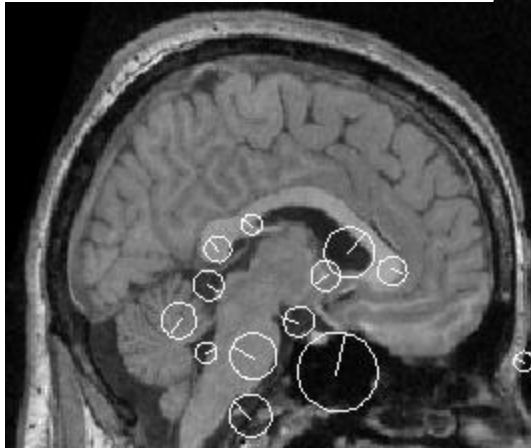


Feature-Based Morphometry: Discovering Group-related Anatomical Patterns.
Toews, Matthew and Wells III, William M., and Collins, D. Louis and Arbel, Tal.
NeuroImage, 2010, Vol 49 (3), 2010, pp. 2318-2327.

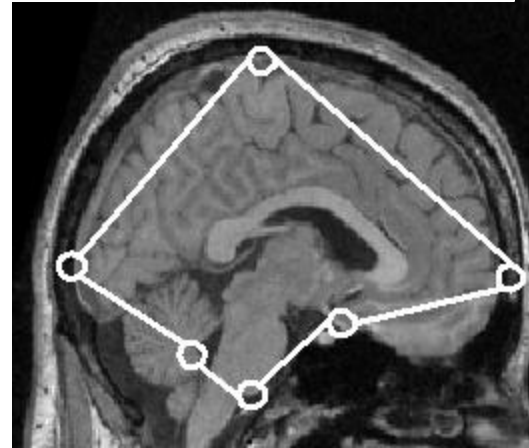
Feature-based Analysis

- Alignment/Registration: fast and robust

Feature-based Alignment
(robust to perturbation)

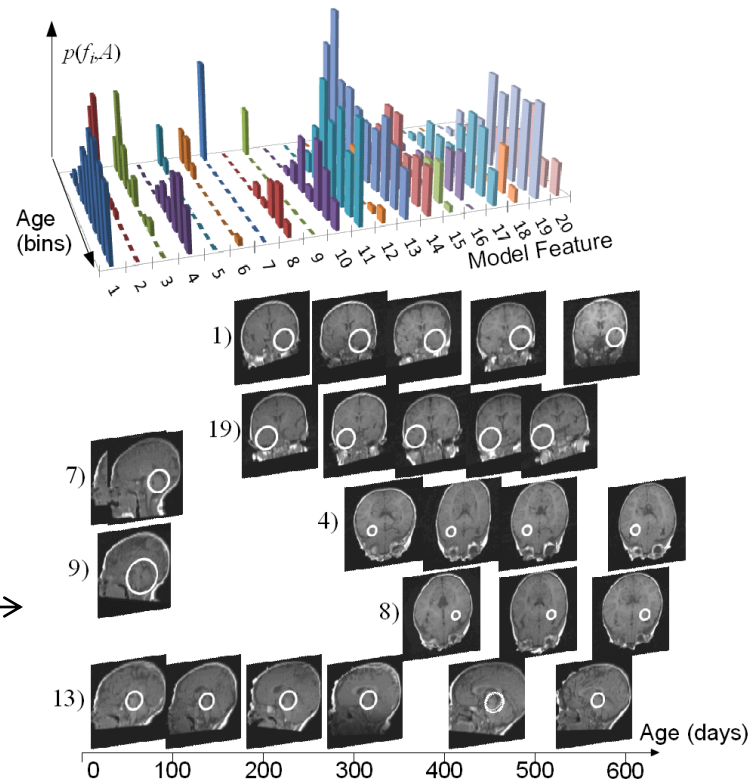


Global Image Alignment
(sensitive to perturbation)



Feature-based Analysis

- Model features over many images.



Infant brain development.

A Feature-based Developmental Model of the Infant Brain in Structural MRI.

M. Toews, W.M. Wells III, Lilla Zöllei.

Medical Image Computing and Computer Assisted Intervention, 2012.